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NOTE-BOOK

ON

PRACTICAL SOLID

OR

DESCRIPTIVE GEOMETRY

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NOTE-BOOK

ON

PRACTICAL SOLID

OR

DESCRIPTIVE GEOMETRY

CONTAINING

PROBLEMS WITH HELP FOR SOLUTIONS.

BY

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PREFACE.

IN teaching a large class, if the method of lecturing and demonstrating from the black-board only is pursued, the more intelligent students have generally to be kept back, from the necessity of frequent repetition, for the sake of the less promising; if the plan of setting problems to each pupil is adopted, the teacher finds a difficulty in giving to each sufficient attention. A judicious combination of both methods is doubtless the best, though this is not always easy of attainment in working with numbers —the use of this book may help in accomplishing it.

It is suggested that at the beginning of a chapter, and in some cases with each problem, the teacher should give a black-board explanation, carefully pointing out any fresh steps, before sending his pupils to their work. The number of examples in each chapter to be worked out by the student is, of course, left to the teacher's judgment of the progress and requirements of his pupil.

The student will here be assisted, it is believed, because he will find help given when necessary, and thus will often be able to go on with his work by himself, with the satisfaction also of feeling that he is progressing systematically. Moreover he will become practised in dealing with written questions—a point not to be lost sight of by those who are preparing for Examinations.

The *Second Edition* has been enlarged by the addition of chapters on the straight line and plane with explanatory diagrams and exercises, on tangent-planes, and on the cases of the spherical triangle.

It is hoped that the work, thus rendered more complete, may prove still more useful as a class-book and means of self-instruction to the various and constantly increasing classes of students for whom it is designed. It was originally intended as an aid in teaching the Mechanical Drawing Class at the Royal School of Mines from Professor Bradley's *Elements of Practical Geometry.* The authors of this work were associated with him in his duties at King's College, London, and the Royal Military Academy, and learnt practically the value of his treatise; but the cost of that work has rendered it inaccessible to many for whom the present book may be available.

A greater number of diagrams have not been added, in order that students may be thrown upon their own resources, and encouraged to consider the *principles* upon which their work proceeds, more than they would probably do if there were figures always at hand for reference.

The *Third Edition* has been revised and considerably enlarged.

Solutions of many Problems set in the Science Examinations—see Chapter on Miscellaneous Problems—and a "Note on the Projection of Shadows" have been kindly supplied by Mr A. G. Meeze, Student of the Royal School of Mines,

January, 1875.

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ELEMENTARY
SOLID OR DESCRIPTIVE GEOMETRY.

ELEMENTARY EXPLANATIONS, DEFINITIONS AND THEOREMS.

> "The object of Descriptive Geometry is the invention of methods by which we may represent upon a plane having only two dimensions, namely length and breadth, the form and position of a body which possesses three dimensions, namely length, breadth, and height."
>
> HALL'S *Elements of Descriptive Geometry.*

IN Descriptive Geometry Solids are represented by their drawings or *projections* on two planes conceived at right angles to one another, intersecting in a line called the *ground line*, *xy*, and named from their usual positions the *horizontal* and *vertical planes of projection*. See Fig. 1.

These planes form four dihedral angles.

Drawings or *projections* on the horizontal plane are called *plans;* on the vertical plane *elevations.*

two planes the *projection* of the line, i.e. its *plan*, or in the case of *AB* and *a'b'* its *elevation*.

The projection of a line may also be defined to be the sum of the projections of its points.

If the given line be a curve, not lying in a single plane, the Superficies containing all its projectors for a given plane of projection, is termed the *projecting surface* of the curve: the intersection of this surface and plane is the *projection* of the curved line.

Hence it will appear that a line is defined or expressed for the purpose of 'Descriptive Geometry,' by the aid of two planes of projection and two *projecting planes* or *projecting surfaces*.

Solid bodies, as they possess three dimensions, length, breadth, and height, require to be represented or projected upon both planes of projection, so that 'height' as well as 'length' and 'breadth' may be exhibited or determined by the drawings.

But as we can have for drawing only one plane, that is, one flat sheet of paper, not two at right angles to each other, the upper part of the vertical plane is supposed to rotate backwards upon *xy*, its intersection with the horizontal plane, as upon a hinge, until it coincides and forms one plane with the horizontal. The plan *a* and elevation *a'* of any point *A will then lie in the same perpendicular to the ground line.*

This perpendicular will thus form a *locus* of points *a* and *a'*, that is to say, must contain points *a* and *a'*.

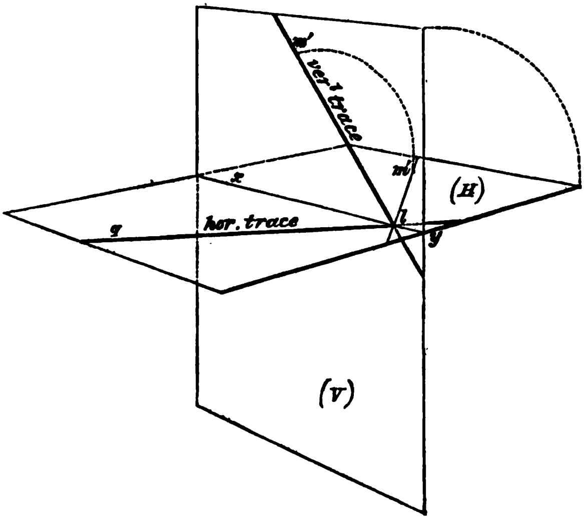
The distance of *a* from *xy* shows the distance of the point *A* from the vertical plane.

The distance of a' from xy shows the distance of the point A from the horizontal plane, i.e. its height.

In all constructions reference is continually made to these relations, which may be thus stated for reference.

Theorem I. The two projections of a point are in the same perpendicular to xy after the vertical plane has been turned into the horizontal.

Fig. 2.



Theorem II. The perpendiculars on xy from the projections of a point are respectively equal to the distances of the point from the planes;

i.e. The perpendicular $a'a$ from the vertical projection a' is equal to the height of the point.

The perpendicular *aa* from its *plan* is equal to the distance of the point from the vertical plane of projection.

The point in which any line or line produced pierces the horizontal plane is called its *horizontal trace:* that in which it pierces the vertical plane its *vertical trace.*

The line in which a plane or surface cuts the horizontal plane is called its *horizontal trace;* that in which it cuts the vertical plane its *vertical trace.* Fig. 2. These two traces intersect in the ground line *xy*, unless by the position of the plane they are parallel to it; or the plane is parallel to one of the planes of projection, in which case there will be only one trace.

Notation.

A point in space is indicated by a capital letter *A*, its projections by *a* and *a′*; the accented letter denoting the vertical projection or elevation of the point.

Similarly a line in space is denoted by the capital letters *AB* and its projections by *ab*, *a′b′*.

A plane is denoted by three letters *hav′*, one on each trace and the third on their point of intersection in *xy*. Figs. 5 and 2.

If two planes be perpendicular to each other, every line drawn from a point in one plane perpendicular to the other lies wholly in the former; and therefore meets the latter in the common section of the planes.

From this and the definitions it follows; that

THEOREM III. (1) A point in one plane of projection has its projection on the other *in xy*;

i.e. If the point be *in* the vertical plane, its *plan* is *in xy*. If in the horizontal plane, its *elevation* is *in* that line.

(2) If a given plane be perpendicular to one plane of projection, its trace on that plane contains the projections on the same of all points, lines, &c. in the given plane.

(α) Thus if an inclined plane be perpendicular to the vertical plane, the elevations of points in the inclined plane will be in its *vertical* trace. Ex. gr. A circle of 3 inches diameter in a plane 30° which is perpendicular to the vertical plane, has a segment of 3 inches of the vertical trace for *its elevation.*

(β) A plane inclined to the vertical plane of projection and perpendicular to the horizontal plane, has the *plans* of all lines in it in its *horizontal* trace. Accordingly the plans of the vertical faces of a right prism resting on its base coincide with the lines forming the sides of that base. See Fig. 5.

THEOREM IV. The projections of parallel straight lines are parallels.

THEOREM V. The projections of a perpendicular to a plane are perpendiculars to the traces of the plane;

i.e. The *plan* of the perpendicular is at right angles to the horizontal trace; the *elevation* of the perpendicular is at right angles to the vertical trace.

THEOREM VI. If a *straight* line lies in a plane and has traces, they will be *in* those of the plane. Hence, if a plane is required which shall contain a given line, the traces of the line must be determined and those of the plane drawn through them.

THEOREM VII. The angle which the tangent plane to a right cone makes with the base of the cone is measured by the plane angle which a generatrix of the cone makes with the base.

THEOREM VIII. If a plane touches a conic surface and a second plane intersects them, the *trace* of the tangent plane on the intersecting one is a tangent to the trace of the conic surface on the same.

THEOREM IX. (α) The orthographic projection, i. e. the plan or elevation, of a finite straight line is equal to the original line if the latter be parallel to the plane of projection. The projection diminishes as the angle of inclination of the line increases, and becomes a minimum, i.e. a point, when the angle of inclination reaches 90°: or when the line is perpendicular to the plane*.

(β) The projection of a rectilineal angle on a plane parallel to that of the angle is an angle equal to the given one.

The projection, on the same plane, of the circular arc subtending that angle will also be equal to that arc. See Figs. 21 and 22.

Similarly, the projection of any figure on a plane parallel to it is a figure equal in all respects to the given one.

---

* The length of the projection of a finite line L inclined θ^0 to the plane of projection is $L \cos \theta$.

And the area of the *projection* of a given figure decreases as the dihedral angle of the plane of projection and the plane of the given figure increases.

When this angle is a right angle the projection of the figure is a minimum, i. e. a straight line, as before indicated.

These considerations will determine the positions of the planes of projection selected, in reference to the data or quæsita of a given problem.

A particular operation called '*constructing*' a plane is often necessary, which we now proceed to explain.

A plane is said to be *constructed* when it has been folded or revolved about its trace as an axis into a plane of projection: carrying with it all points, lines and figures, which are contained in it.

The plane may thus revolve about its vertical or horizontal trace into the corresponding plane of projection.

This operation is necessary when it is required to show the real magnitude of an angle or figure lying in a single plane.

THEOREM X. Points so 'constructed' will lie or have their *loci* in perpendiculars to the trace used, drawn through their corresponding projections.

Thus if the *horizontal* trace be used the 'constructed' points will lie in perpendiculars to that trace through their respective *plans*, &c.

The reason for this relation or 'locus' will be obvious by considering that perpendiculars on the trace from points in the plane move in planes perpendicular to that axis.

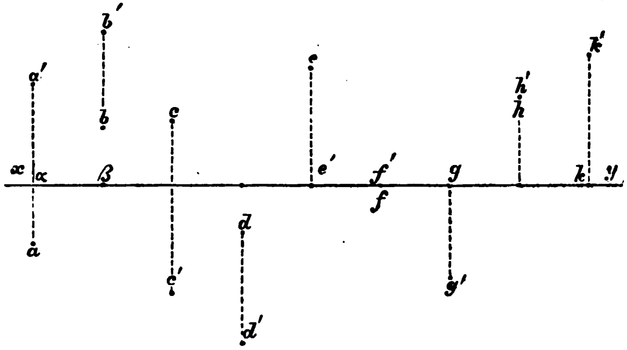
Also if points have *fixed* positions with regard to the rotating plane, their projections upon it when in its second position will be in perpendiculars to the trace used, drawn through the elevations or plans of the points according as that trace is the horizontal or vertical one.

These loci are to be made use of in determining the projection of a frustum of a solid on a plane which passes through it.

DEFINITIONS ILLUSTRATED.

Fig. 3.

Points.



Examples of points determined in various positions by their projections.

Point *A* or *aa'* is in the 1st dihedral angle of the planes of projection.

„ *B* or *bb'* „ 2nd „ „ „ „ „
„ *C* or *cc'* „ 3rd „ „ „ „ „
„ *D* or *dd'* „ 4th „ „ „ „ „
„ *E* or *ee'* is in the horizontal and behind the vertical plane.
„ *F* or *ff'* is in both planes.
„ *G* or *gg'* is in the vertical and below the horizontal plane.

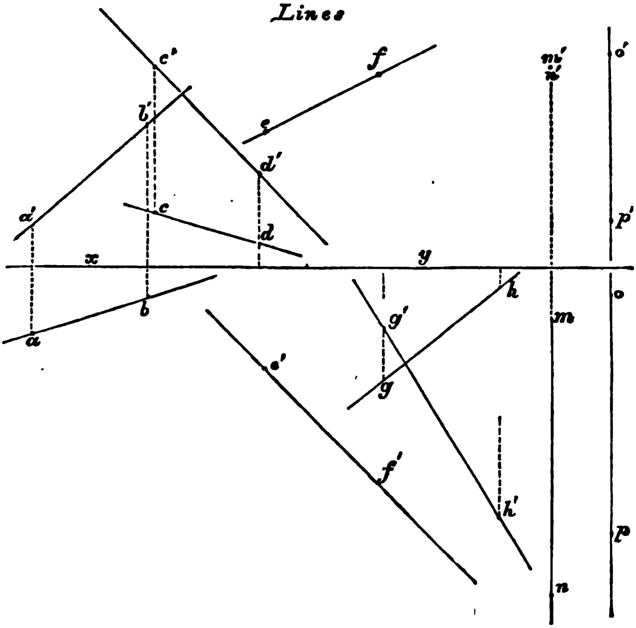
Point *H* or *hh'* is equidistant from the horizontal and vertical planes.

,, *K* or *kk'* is in the vertical and above the horizontal plane.

Observe. That to find the projections of points satisfying given conditions of position with regard to the planes of projection the relations indicated in Theorems I. and II. will suffice.

Fig. 4.

Lines

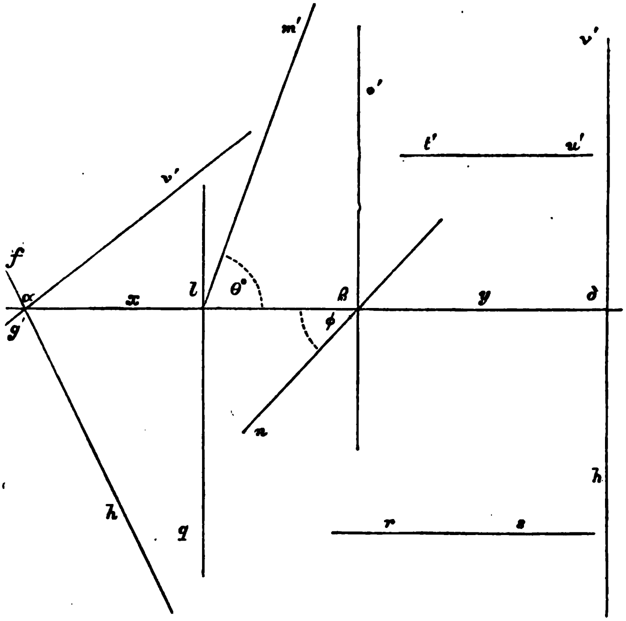


Examples of lines in various positions determined by their projections.

The segment AB or ab, $a'b'$ is in the 1st dihedral angle of planes.
" CD or cd, $c'd'$ " 2nd " "
" EF or ef, $e'f'$ " 3rd " "
" GH or gh, $g'h'$ " 4th " "
" MN or mn, $m'n'$ " 1st " "
and the line is perpendicular to the vertical plane.
" OP or op, $o'p'$ is in the 1st dihedral angle.

Fig. 5.

Planes



Examples of planes defined or expressed by their traces.

The portion included between the infinite branches ah and av' of the given plane is in the first dihedral angle of the planes of projection.

The portion between the branches av' and af is in the second dihedral angle.

The portion between af and ag' is in the third angle, and the portion between branches ag' and ah is in the fourth angle.

The plane qlm' is perpendicular to the vertical plane and makes an angle of θ° with the horizontal.

The plane $n\beta o$ is at right angles to the horizontal and makes an angle of ϕ° with the vertical plane.

The plane rs, $t'u'$ is parallel to xy and meets both planes.

The plane $h\delta v'$ is at right angles to xy and therefore to both planes of projection.

MEMORANDA

FOR WORKING THE PROBLEMS IN ENSUING CHAPTERS.

All the solids given are assumed to be '*right,*' unless otherwise expressed.

All dimensions and arrangements, not expressly mentioned and limited, may be assumed at pleasure.

The inclinations of all lines and planes must be understood, unless otherwise mentioned, to be *to the horizontal plane.* So likewise with all '*constructions;*' see page 8.

Invisible edges. In *plan* the invisible edges are those which are hidden by the rest of the solid to the eye looking through the solid at right angles to *the horizontal plane.* In *elevation* they are those which are similarly concealed from the eye looking through the solid at right angles to *the vertical plane.* Invisible edges must be drawn in dotted lines.

Sections. The teacher should give directions for sections with each problem, as far as possible.

Also several *plans* and *elevations* may advantageously be completed of the solids given in many of the problems.

I.

SOLIDS IN SIMPLE POSITIONS.

1. *Draw plan and elevation of a pyramid,* 3·5 *inches high, with square base of* 2·5 *inches side, when resting with its base on the horizontal plane, and with one side of the base making an angle of* 30° *with the vertical plane.*

Commence with the *plan.* This will be the square of the base with the opposite corners joined for the plans of the slant edges of the solid.

For the *elevation* take the ground line xy inclined 30° with one side of the square and draw perpendiculars to xy (Theorem I.) from the four corners and the centre of the square. The elevations of the four corners of the square will be in the ground line, because the base of the solid rests on the horizontal plane. The height of the pyramid, 3·5 inches, must be set up from the point where the perpendicular from the centre of the square meets the ground line, and at right angles to the ground line (Theorem II.). The whole elevation will be completed by joining the elevation of the vertex to the four points determined on the ground line for the elevations of the four corners of the base.

Invisible edges. In this *plan* all the edges of the solid are visible. In this *elevation* the slant edge of the pyramid nearest *xy* is invisible. The two sides of the base nearest *xy* are also invisible, but as they coincide with the two other sides in the *xy*, they cannot be shown in dotted lines.

NOTE. It must be understood that from any one *plan and elevation* any number of elevations can be drawn by assuming vertical planes, that is, by taking new ground lines, in different positions round the solid, and working from the *plan* to them. For example, if a solid were placed on the floor of a room, four elevations of it might be shown on the four walls of the room, which are so many vertical planes of elevation. And so from any one *elevation and plan* any number of plans may be determined by assuming horizontal planes in the desired position about the elevation, and working from the *elevation* to them.

E.g. *For a new elevation.* The new ground line, *xy*, being taken, draw perpendiculars to it from the *plans* of the various points of the solid, and on these perpendiculars from *xy* mark the various heights of the points, to be taken from the elevation already drawn, and complete by joining the points thus found as already joined in plan.

For a new plan. The new *xy* being taken, draw perpendiculars to it from the *elevations* of the various points of the solid, and on these perpendiculars mark from *xy* the various distances which the points are from the first plane of elevation to the *plan* first drawn, and complete by joining the points as already joined in elevation.

2. *Plan and elevation of a pyramid with base a regular hexagon of* 1·5 *in. side, when resting on its base, and with one side of base inclined* 40° *to the vertical plane. Height* 3·5 *inches.*

3. *Plan and elevation of a prism, 3·5 inches long, with square bases of 2·5 inches side, when resting on one base and with one face inclined 30° to the vertical plane.*

From its position a square of 2·5 inches will represent the whole solid in *plan*. The sides of the square will be the plans of the vertical faces; if therefore *xy* be taken at 30° with one of these sides, the required position of the vertical plane will be secured.

4. *Plan and elevation of a cylinder; axis 3·5 inches, horizontal, and inclined 40° to the vertical plane. Diameter of base 2·5 inches.*

The *plan* of the cylinder will be a rectangular parallelogram. In *elevation* the bases will be two ellipses, which will be best drawn by determining their major and minor axes from the plan. To do this, determine the elevations of the four points which are at the extremities of the horizontal and vertical diameters of each circular base: these points will be the extremities of the major and minor axes of the required ellipses.

5. *Plan and elevation of a cone standing on its base. Height 3·5 inches. Diameter of base 2·5 inches. Determine also a second plan when resting on its conical surface.*

For the second *plan*, use the elevation already determined and draw the new *xy* parallel to one of the sides of the triangle which is the elevation of the cone.

6. *Plan and elevation of a prism, 3·5 inches long, with bases regular hexagons of 1·5 inches side, when its lowest face is inclined 20°, its axis horizontal, and inclined 40° to the vertical plane.*

Commence by drawing an *elevation*, when the solid is so placed that the elevations of both bases coincide, i. e. when the axis is at right angles to the plane of elevation. This elevation will be simply the regular hexagon. A *plan* can readily be deduced from it by taking a ground line making an angle of 20° with one of the sides of the hexagon. A second *elevation*, the one required in the question, can then be determined by taking another ground line at 40° with the plan of the axis. The heights in this second elevation can be taken from the first auxiliary elevation.

7. *Plan and elevation of the prism, Problem* 3, *with its axis horizontal, inclined* 45° *to the vertical plane, and its lowest face inclined* 30°.

The work corresponds to that in the last Problem.

8. *Plan and elevation of a tetrahedron with one face horizontal and* 1 *inch above the horizontal plane. Edge* 3 *inches. One horizontal edge to be inclined* 15° *to the vertical plane.*

The *plan* will be an equilateral triangle of 3 inches side, with the corners joined to the centre for the plans of the slant edges of the solid.

The base being horizontal, its elevation will be a straight line and at the required height, 1 inch, above the ground line.

The edge of the tetrahedron being known, the height of the vertex above the horizontal base can be determined by making a vertical section through the vertex and one slant edge. To find this, make a right-angled triangle with the length from a corner of the base to its centre for the base of the triangle, and the true length of the edge of the tetrahedron for the hypothenuse. The perpendicular will be the height required.

9. *The surface of a sphere, diameter 3 feet, is divided into equal portions by 4 great circles, one horizontal and three vertical. Draw an elevation of the sphere and circles on a plane parallel to one of them. Scale $\frac{1}{12}$.*

10. *A street-lamp is formed of a right pyramid, and a frustum of one; the slant edges of former inclined 45°. The common base of the solids is a square of 20 inches side, and the lower base or end of frustum is a square of 12 inches side and 20 inches below the other.*

(*a*) *Determine an elevation on a plane parallel to a horizontal edge.*

(*b*) *Determine an elevation on a plane which makes an angle of 30° with a horizontal edge.*

Scale $\frac{1}{8}$.

The *plan* is a square of 2·5 in. with the opposite corners joined for the slant edges, and with a second square of 1·5 in. within and parallel to the first.

In the *elevation* (*a*) the slant edges are drawn 45° with the elevation of the base of pyramid.

11. *A ladder 50 feet long and of the uniform width of 3 feet, rests against a vertical wall and makes an angle of 70° with the horizontal plane on which it stands.*

(*a*) *Draw a plan, with an elevation on a plane parallel to that of the wall.*

(*b*) *Also an elevation on a plane making an angle of* 40° *with that of the wall.*

Scale 10 *feet to* 1 *inch.*

N.B.—*Rungs and sides to be represented each by a single line. Three rungs only to be shown, dividing the length of the ladder equally.*

Commence with an auxiliary elevation on a vertical plane parallel to the ladder.

For this, draw xy and lay off the angle 70°, and on this line measure the length of the ladder: thence determine plan.

II.

COMBINATIONS AND GROUPS OF SOLIDS.

1. *A prism, with bases equilateral triangles, is placed with one face on the horizontal plane. On the centre of it rests a brick,* 9 *by* 3 *by* 4·5 *inches, with one of the short edges of a large face touching the ground, with the centre of that face touching the highest edge of the prism and with the face itself inclined* 45°. *Horizontal edge of prism* 9 *inches. The side of the equilateral base to be determined from the question. Scale* ⅛. *Draw plan and elevation, and a second elevation taking the vertical plane at* 30° *with any horizontal edge of the prism.*

Commence with an *elevation* so placed that both bases of the prism coincide. To do this, draw first the elevation of the brick, which will be simply a rectangular parallelogram. Through the lowest corner take xy making the required angle with the line that represents the face inclined 45°, and make the centre of this line the vertex of the triangle, recollecting that the elevation of the base of that triangle must according to the conditions be in xy, the ground line. The work for the *plan* is simple. The second elevation will be determined by taking the new ground line 30° with any horizontal edge of the prism, and working to it.

2. *Instead of the prism take a cylinder with brick touching it and otherwise disposed as above. The base of the cylinder to be determined from the question;* i.e. *so that the horizontal bisecting line of the face of the brick may be in contact with the cylinder.*

3. *Plan and elevation of a block and prism under the following conditions:*

Block, 1 *inch thick, with faces* 3 *inches square.*

Prism, base a regular hexagon of 1 *inch side, edges of the faces* 4 *inches, extending equally on each side of the block and at right angles to its square faces, axis of prism passing through the centre of the block, two faces of prism parallel to two narrow faces of the block.*

(1) *Plan and elevation when the prism has two faces horizontal, the block standing with one narrow face on the horizontal plane and with the short edges of that face inclined* 40° *to the vertical plane.*

Commence with an elevation so placed that both bases of the prism coincide. The elevation of the block will then be the square, and that of the prism the regular hexagon in the centre of this square with two sides parallel to two sides of the square. From this auxiliary elevation determine plan, and thence required elevation.

(2) *Plan and elevation when the whole is tilted so as to rest on one edge of the block and on one corner of the base of the prism.*

If a ground line is taken touching one corner of the block and one adjacent corner of the prism in the *plan* determined above, this plan may be considered as an *elevation* of the solids in the required position and the required plan be at once deduced therefrom by working to the ground line so taken.

4. *Plan and elevation of a cylinder,* 4 *inches long, diameter of base* 2 *inches, passing through the centre of a circular slab,* 1 *inch thick and* 3 *inches in diameter; the axis of the cylinder at right angles to and extending equally from both faces of the slab. The whole to be placed resting on the rim of the slab and the rim of the cylinder.*

5. *Describe three circles with radii* 1·5 *in. touching one another and join their centres. Consider the circles as the plans of three spheres resting on the horizontal plane, and the triangle as the plan of the base of a tetrahedron resting upon them. Complete plan and elevation of the whole, and draw a second plan when one of the slant edges of the tetrahedron is horizontal.*

For the second plan take xy parallel to the elevation of any slant edge of the tetrahedron.

6. *A tetrahedron, edge* 2·5 *inches, stands on a square block, and has an edge parallel to a side of the square. The block is* 1 *inch thick, and its square faces have sides of* 3·5 *inches. An axis of the tetrahedron if produced, passes through the centres of the squares.*

Draw plan and elevation—

(*a*) *When the square faces of block are horizontal and the plane of elevation makes* 30° *with a side of a square.*

(*b*) *When only one slant edge of tetrahedron is horizontal.*

7. *A circular slab, 1 inch thick, diameter 3 inches, rests on one face. On it is placed a tetrahedron of 2 inches edge, axis vertical and over the centre of the slab. Draw plan and elevation, and a second plan when the rim of the slab and the vertex of the tetrahedron touch the horizontal plane.*

For the second plan take *xy* touching the elevations of the vertex and the rim of the slab.

8. *A rectangular prism, 4 inches long, with square bases of 2·5 inches side, is to be drawn when one diagonal of the solid is either vertical or horizontal.*

Draw a plan and elevation of the solid resting on one of the square bases, with one diagonal of the square at right angles to the ground line.

If two opposite corners of the rectangle which is the elevation of the solid be joined, this line will be the elevation of the *diagonal of the solid* shown from its position in its true length. A plan may then be determined with this diagonal vertical *or* horizontal by taking *xy* at right angles to *or* parallel with the elevation of this diagonal.

9. *Plan and elevation of a pyramid with hexagonal base.*

a. *With one face* ABV *horizontal.*
b. *When one face is vertical.*
c. *When one edge is horizontal.*
d. *When one edge is vertical.*

Determine plan of the pyramid when resting on its base.

For *a.* Determine elevation on a vertical plane taken at right angles with the plane of face *ABV*. Then the required plan may be deduced by taking *xy* parallel to the line which represents the elevation of face *ABV*, and working to it.

For *b*. Take *xy* perpendicular to the line representing the elevation of face *ABV*.

For *c* and *d*. Determine an elevation where the edge is shown in its true length. Then *xy* taken parallel for *c*, perpendicular for *d*, to the elevation of this edge will place the pyramid in proper position for the plans required.

10. *Plan and elevation of an octahedron, edge 3 inches.*

(*a*) *When an axis of the solid is vertical and rests on the horizontal plane.*

(*b*) *When resting with one face on the horizontal plane.*

(*a*) Describe a square of 3 inches side and join the diagonals. This will be a *plan* of the solid in required position. For the *elevation* it is only necessary to know that the length of the axis is equal to a diagonal of this square, and that the heights of the horizontal edges are half that diagonal.

(*b*) Describe an equilateral triangle of 3 inches side, and from its centre with radius to one of the corners describe a circle. In this circle complete the hexagon, of which the three corners of the triangle are alternate points. This hexagon with the alternate points duly joined is the required *plan*.

For the *elevation* it is necessary to find the distance between the parallel faces. This is done by a vertical section through one axis of the solid. This section is a rhombus, the shorter diagonal of which is an edge of the solid, and the side of which is the altitude of the equilateral triangle forming the horizontal or any other face of the solid. The distance between the opposite sides of the rhombus gives the distance between the parallel faces of the solid.

III.

PROBLEMS ON THE STRAIGHT LINE AND PLANE.

Obs. **To express** points, lines and planes, see Problems 1 to 7.

PROBLEM I.

Given the projections ab, a'b' *of a finite straight line to determine:*

1. *The length of the segment* AB *of the given line, or the distance between points* A *and* B.

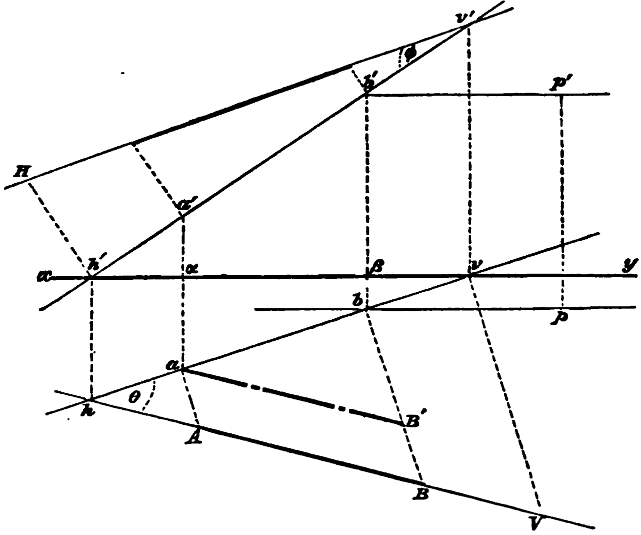
2. *The angle of inclination of the line to each plane of projection.*

3. *The traces of the given line and the distance between them.*

1. The line *AB* lies in each of its projecting planes: 'construct' or turn either of these into the corresponding plane of projection and the segment *AB* is shown.

Let the projecting plane for the plan *ab* revolve about this line as an axis, into the horizontal plane.

Fig. 6.



From points a and b set off the projectors aA and bB, equal to $\alpha a'$ and $\beta b'$ respectively, and at right angles to ab, on the same side of it.

Join A and B; AB is the required distance or length of segment.

Observe. The length AB (being the hypothenuse of the right-angled triangle of which the sides are ab and the difference of the projectors Bb and Aa) may be found by setting this difference from b along bB to B' and joining B' to a.

2. The angle a straight line makes with a given plane is the angle between the line and its orthographic projection on that plane, or the angle which AB makes with ab, and is therefore the angle θ.

For the same reason ϕ is the angle which the given line makes with the vertical plane of projection.

3. The 'traces' of the line being points *in* the planes of projection have each of them a projection in xy, viz. h' and v (Theorem III.); but the elevation $a'b'$ produced contains h', therefore the intersection of these lines, namely, xy and the elevation, gives point h'; and Theorem I. and the plan of the line give point h. The corresponding loci of v and v' determine those points.

The distance between the traces H and V of the given line is obviously the length hV, as shown in figure.

Corollaries given below.

As an exercise on this problem determine the inclinations of the given lines, AB, CD, EF, &c., and their traces, and the lengths of the segments indicated. See Fig. 4.

Converse of Problem 1:

(1) Given the traces of a straight line; or

(2) Its inclination and plan; or

(3) Its length and inclination; or

(4) Length and position of extremities of finite portion; to draw or determine the projections of the line.

(1) If the traces hh' and vv' be given,—

Join h and v and h' and v'.

Then hv, $h'v'$ are the projections required.

(2) If the plan hv be given making a given angle $hv\alpha$ with xy, and the line inclined at an angle of θ degrees.

At any point h in the given plan draw hV, making the angle θ° with hv.

Assume points a and b in the plan, draw the perpendiculars aA and bB, and find points a' and b' by Theorems I. and II. and join a', b'.

Then ab, $a'b'$ are the projections.

(3) If the length AB be given, its inclination θ, and the position aa' of one extremity A.

Through a draw the indefinite plan hv.

Set aA at right angles to ab and equal to aa'.

Through A draw hV, making the angle θ with hv.

From A set off AB along this line and determine b and b'.

ab, $a'b'$ are the required projections.

(4) If the length AB be given and the positions of its extremities A and B with regard to the two planes.

Draw aa' the projections of one extremity A, from the conditions Theorems I. and II.

Also any point pp' from the conditions for the other extremity B and through p draw a parallel to xy: this is a locus of b the other extremity of the plan ab required. Determine the length ab* of the plan; measure this length from a to a point b in the parallel, and the position of ab is fixed; b' may be found of the height of p', and ab, $a'b'$ are determined.

The student will observe that the problems admit of one, two, or more, or an infinite number of solutions. These cases should be studied. See Problem 2.

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* The base of a right-angled triangle, hypothenuse AB and altitude equal the difference of the heights of A and B.

I. (3), COR. 1. *To determine the 'traces' of a given plane when the data are:*

(1) *Two straight lines (parallel or meeting).*

(2) *A straight line and a point.*

(3) *Three points.*

(1) The horizontal and vertical traces of the given lines must be found by Problem 1, and the corresponding traces of the plane drawn through the points thus found. Theorem VI.

(2) Find the traces of the given line.

Draw the projections of a line parallel to it, through those of the given point (Theorem IV.), and find its traces.

Or, by Theorem I., determine the projections of any point qq' *in* the given straight line; join the projections of this point with pp' those of the given one; then the line PQ lies in the given plane.

The traces of the two lines give those of the plane.

(3) Join the corresponding projections of the given points; and determine the traces of the three lines of which those joining lines are the projections.

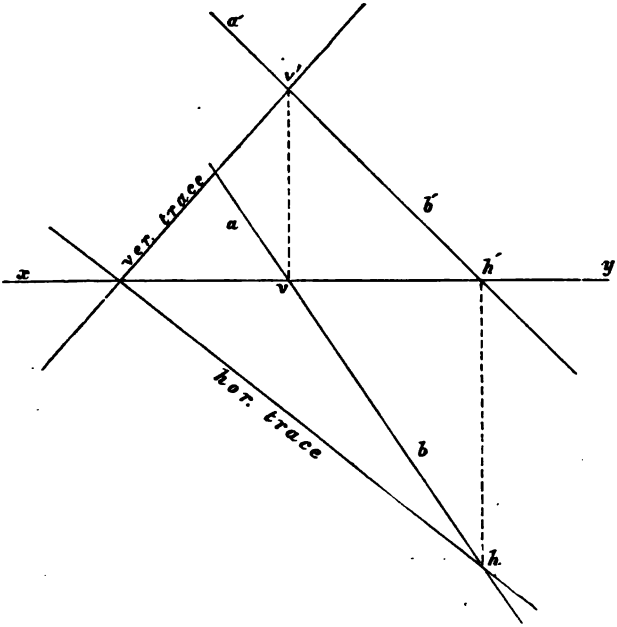
The traces of the three lines give those of the plane.

Obs. If the given plane meets xy only three traces of the lines lying in it need be found; e.g. two in the horizontal trace of the plane and one in its vertical trace: the latter joined to the point of intersection of the horizontal trace of the plane with xy gives the vertical trace of the plane.

I. COR. 2 (see figure). *Given the traces of a plane and one projection* ab *of a straight line which lies in that plane; to determine the second projection* a'b' *of the line.*

Since the traces *hh'* and *vv'* of the line must be *in* those of the given plane (Theorem VI.), determine these points, then the line drawn through points *h'*, *v'* is *a'b'* the required elevation.

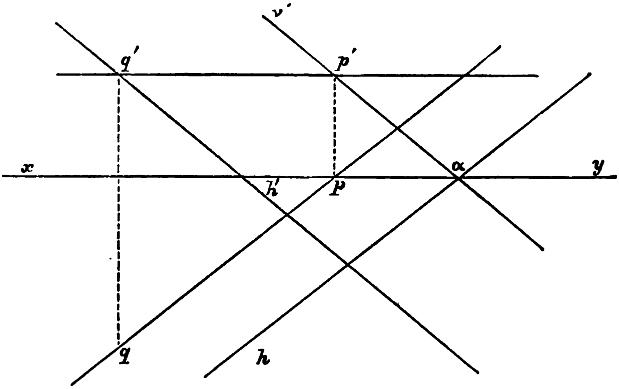
Fig. 7.



Observe. Point *H* lies in the given straight line *AB*, hence the projections *hh'* of the point are in those of the line. Thus *ab* is a locus of *h*, and of *v*; the horizontal trace of the plane is a second locus of *h*, and *xy* of *v*, which points are thus determined: perpendiculars to *xy* through them are loci of *h'* and *v'*, Theorem I., which points are thus also determined.

I. Cor. 3. *To determine a point* pp′ *of given altitude* a *inches* (*or* a″) *in a given plane* hav′.

Fig. 8.



In the given vertical trace *av′*, take a point *p′*, *a″* high, Theorem II.; find *p* in *xy*, Theorem I.; then *pp′* is the required point.

I. Cor. 4. *To determine a horizontal line* a″ *high*, *in a given plane* hav′. Fig. 8.

Determine *pp′* as before.

Through *p* draw *pq* parallel to *ha*, Theorem IV.

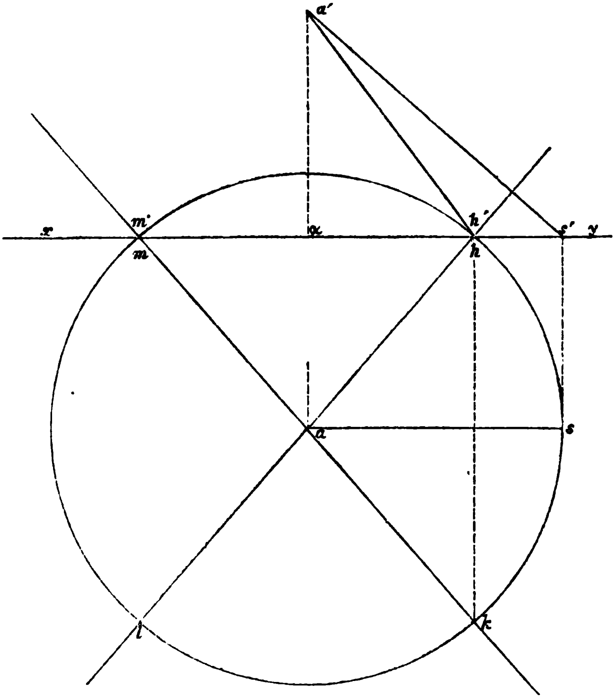
Through *p′* draw *p′q′* parallel to *xy*.

Then *pq*, *p′q′* is the required line.

PROBLEM II.

Through a given point aa′ *to draw a straight line inclined* θ° *to the horizontal plane.*

Fig. 9.



If a right cone be determined having its vertex in the given point A, axis equal to $a'\alpha$ and generatrix inclined θ, every generatrix of the cone fulfils the given conditions. The conic surface is therefore a locus of the required line.

To work the problem:

Through point a' draw $a's'$ making the angle $a's'a$ equal to θ.

Through a draw as parallel to xy, then $as, a's'$ is parallel to the vertical plane and inclined θ°.

With centre a and radius equal to as' describe the circular base klm of the cone which gives the complete solution.

Observe. The four lines AH, AK, AL, AM are inclined θ°, and make equal angles with the vertical plane.

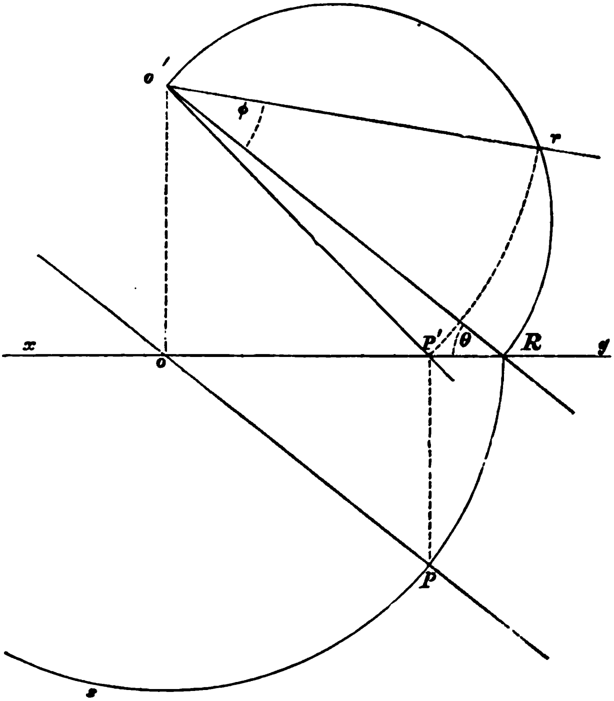
PROBLEM III.

To determine a straight line which shall make a given angle θ with the horizontal plane and an angle ϕ with the vertical plane of projection.

A line inclined θ° will lie on the surface of a right cone with axis vertical and generatrix inclined θ° to the base.

Determine such a cone having its vertex in point oo' in the plane of elevation: Rps will be its circular base. That generatrix of the cone which makes an angle ϕ with the vertical plane will be the line required. To find this. Knowing the length $o'R$ of this generatrix, determine the length $o'r$ of its elevation as in the figure; and since the position of o' one extremity of it is fixed, that of the other, viz. p', in xy, may be found as indicated. The perpendicular to xy from p' is a locus of p, and intersects the circular arc *Rps*, which is another locus of *p*, which point is thus determined. Join *op*, *o'p'*; it is the required line.

Fig. 10.



It is obvious that there are four such lines on the surface of the cone, and therefore there are four solutions of the problem, when *oo'* is a given point in the line.

It will also appear from the above, that the second angle ϕ cannot exceed the complement of θ or $(90 - \theta)$ degrees.

PROBLEM IV.

To draw or determine the traces of a plane inclined θ degrees and perpendicular to the vertical plane of projection. See Fig. 5, 'Planes'.

Draw the horizontal trace *ql* of the required plane at right angles to *xy*, meeting it in point *l*; and through *l* draw *lm′* making the angle *ylm′* equal θ: *qlm′* is the required plane.

It is obvious that *lm′* and *ly* are the two perpendiculars to *ql*, the horizontal trace, or common section of the planes, at the point *l* in it, which form the measuring-angle of the inclination of these planes to each other.

The plane of *lm′* and *ly* having been turned into the horizontal plane, *lm′* in Fig. is no longer perpendicular to *ql*.

Similarly (see Fig. 'Planes') a plane $n\beta o'$ may be drawn at right angles to the horizontal and making an angle ϕ with the vertical plane of projection.

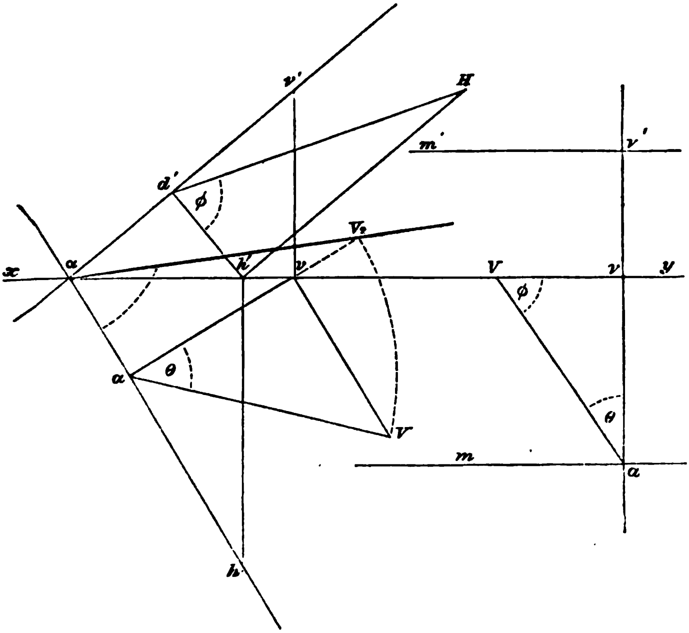
PROBLEM V.

To determine the angles θ and ϕ which a given plane hαv′ *or* am, m′v′ *makes with the planes of projection; and to determine the angle between its traces.*

Since a dihedral angle is measured by the plane angle formed by two perpendiculars drawn one in each plane to the same point in their common section, the plane of this rectilineal angle is perpendicular to the 'trace' of the given plane. 'Construct' the plane of this angle, revolving it about that line of it which is *in* the plane of projection in question, and the required angle of inclination is obtained.

Thus at point *a* in the horizontal trace *ha* (Fig. 11) two such perpendiculars *av* and *aV* were found, and at point *d'* in the vertical trace perpendiculars *d'h'* and *d'H*, and the required angles shown.

Fig. 11.



To determine the magnitude of the angle *hav'* between the traces of the plane.

Let the plane be 'constructed' or folded about its horizontal trace *ha* until it coincides with the horizontal plane.

Then the length aV measured from a along av produced to V_2 gives a second point in the second line of the required angle. Join aV_2, and haV_2 is the angle between the traces of the plane.

PROBLEM VI.

Given the horizontal trace ah *of a plane inclined* θ *to determine its vertical trace* av′.

At any point a in the horizontal trace draw av at right angles to it and meeting xy in v. Fig. 11. Draw aV making the angle vaV equal to the given angle θ. From v draw vV at right angles to av, and from v set off the length vv' at right angles to xy and equal to vV. Then the straight line through a and v' is the vertical trace required.

See Problem III. Chap. VIII.

PROBLEM VII.

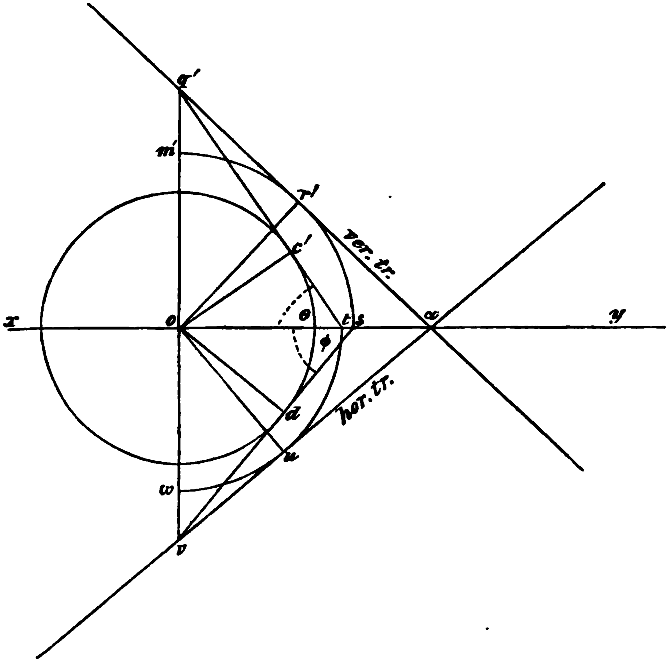
To determine the traces of a plane making an angle of θ *degrees with the horizontal and an angle of* ϕ *degrees with the vertical plane.*

When a plane touches a right cone it is inclined at the angle which the generatrix of the cone makes with its base.

Determine two right cones (Fig. 12) with vertices q' and v and bases tuw and $sr'm'$ in the horizontal and vertical planes respectively; the generatrix $q't$ making an angle θ with its base tuw, and generatrix vs an angle ϕ with its base $sr'm'$. The common tangent plane to these cones is the plane required.

The surfaces of the cones have a common point pp' which is also on the surface of a common sphere enveloped by them and having its centre *o* in *xy*. See Figure 13. Hence the plane of points *P*, *Q* and *V* is that required.

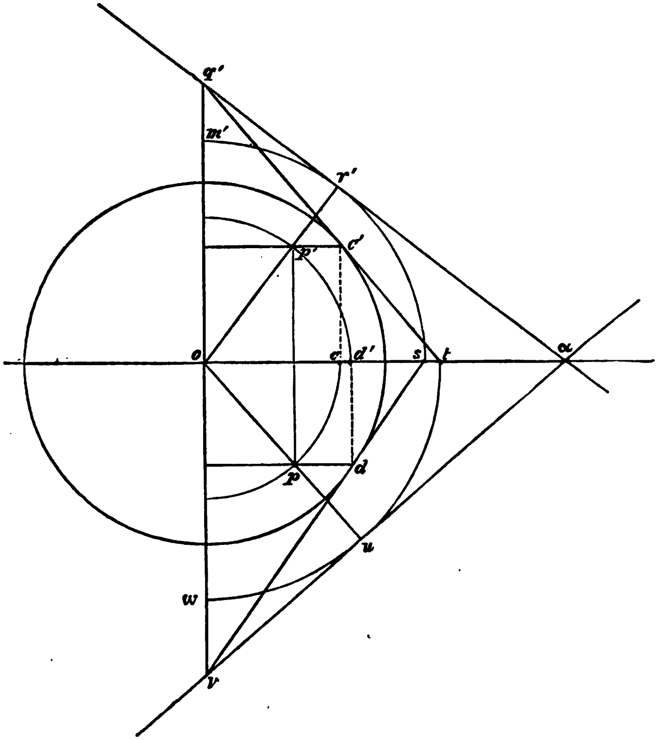
Fig. 12.



To work the problem.

Commence with one of the vertices, assuming point *Q* or *q′* in the vertical plane. Fig. 12. Draw *q′o* indefinitely at right angles to *xy*, meeting it in *o*; and draw *q′t* as above described, making angle *θ* with *xy*, on which let fall the perpendicular *oc′* from *o*; the circle described with centre *o* and radius *oc′* gives the common inscribed sphere of the two cones. *vs* drawn as above mentioned, making the angle ϕ with *xy*, and touching this circle in *d*, meets *q'o* produced in the point *v*, which is the second vertex. Tangents to the bases of these cones through *v* and *q'* meet *xy* in point *a* and give the traces of the required plane *q'av*.

Fig. 13.



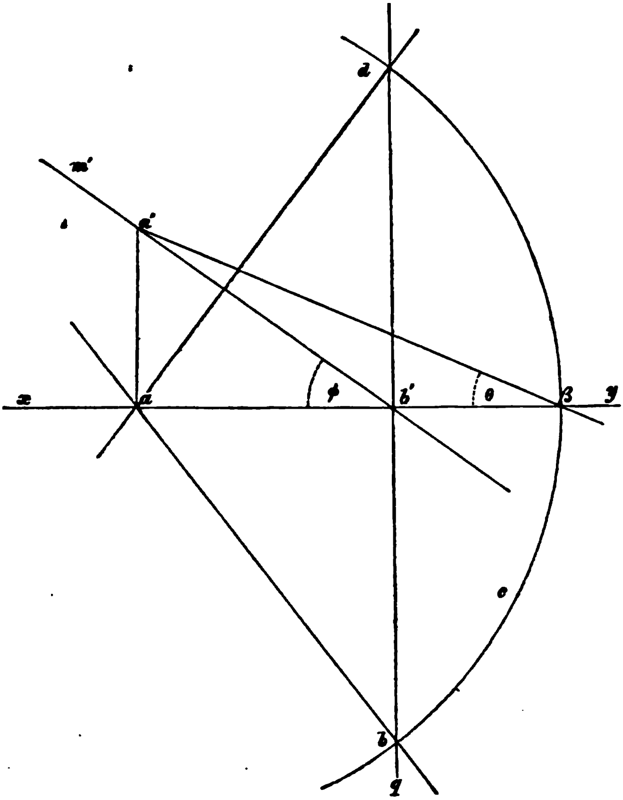
Limits. The sum of the given inclinations or $\theta + \phi$ lies between 90° and 180°.

Obs. To express given combinations of points, lines, and planes, see Problems 8 to 28.

PROBLEM VIII.

To determine a straight line inclined $\theta°$ to lie in a given plane qb′m′.

Fig. 14.



Limits. The angle ϕ is the angle of inclination of the given plane. It is therefore obvious that the angle of inclination θ of the given line must be *not greater than* ϕ. For no straight line can lie in a plane and have a greater inclination than that of the plane.

Construction. Assume point aa' in the given plane.

Make the angle $a'\beta a$ equal θ.

With centre a and radius $a\beta$ describe the arc βcb meeting the trace qb' in b or d; then ab, $a'b'$ or ad, $a'd'$ is the required line.

Observe, that each of the lines AB and AD satisfies the conditions of the problem.

The cone, vertex A and base dcb, is the locus of all lines through A inclined θ.

The given plane is a second locus of the required line. And these loci intersect in the lines AB and AD only.

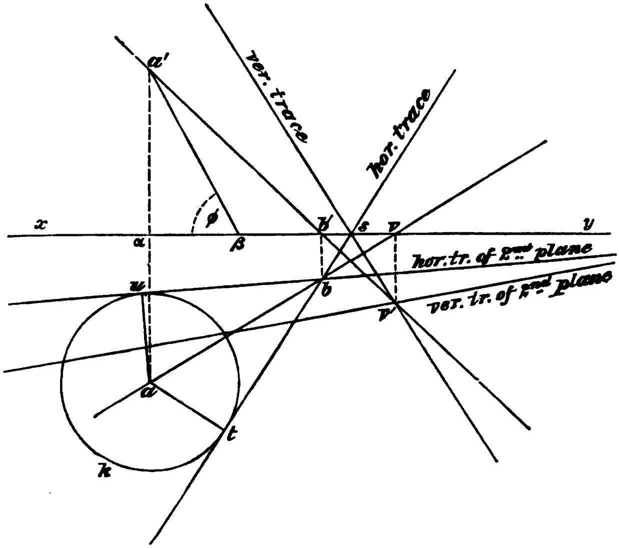
PROBLEM IX (*converse of Problem VIII*).

Through the given straight line ab, a'b' (*which is inclined* θ) *to draw a plane inclined* ϕ^0.

Since the given line is to lie in the required plane, its traces must lie in those of the plane, Theorem VI. Determine these traces bb', vv'. Problem I.

Determine a right cone, with vertex in point aa' of the given line, and base tuk in the horizontal plane, its generatrix making the given angle ϕ with the base. Every tangent plane to this cone is inclined ϕ^0, Theorem VII.; and the straight line through b touching the base of the

Fig. 15.



cone in *t* is the horizontal trace of such plane, Theorem VIII. By producing this line to meet *xy* in *s* and joining *sv'*, we obtain the vertical trace of the required plane.

For this plane *tsv'*, passing through points *A* and *B* in the given line, contains that line; and touching the cone has the required inclination.

Note: That the radius *at* of the base of the cone is taken equal to *αβ* in figure.

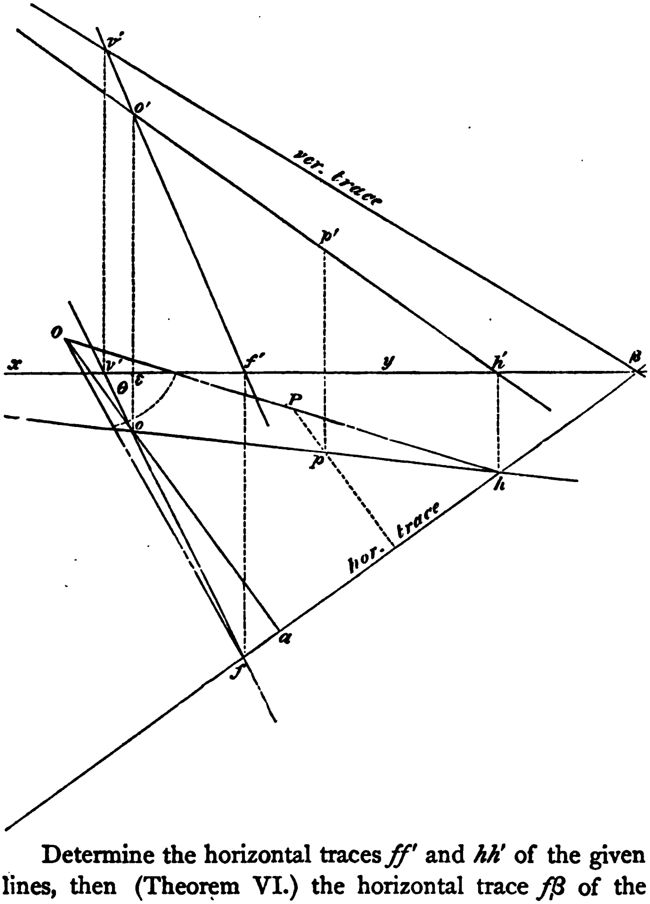
Also, that a second plane *bwv'*, having *ub* for its horizontal trace, likewise satisfies the conditions of the problem.

N.B. Point *w* lies out of the figure.

PROBLEM X.

To determine the magnitude of the angle which the two given straight lines OH, OF *make with each other.*

Fig. 16.



Determine the horizontal traces ff' and hh' of the given lines, then (Theorem VI.) the horizontal trace $f\beta$ of the plane of the lines passes through points *f* and *h*, and its vertical trace *βv'* through *v'*, *vv'* being the vertical trace of *OF*.

'Construct' the plane *fβv'*, revolving it about its trace *fβ* into the horizontal plane; then *fOh* the required angle is exhibited.

The indefinite perpendicular on the horizontal trace through the point *o* is a locus of *O* (Theorem X.), and the length *Oa* is made equal to the hypothenuse of the right-angled triangle, of which *oa* is the base and *o'ε* the altitude.

Cor. *To bisect the angle formed by two given straight lines, as* OF *and* OH.

'Construct' the plane of the angle as above; and bisect the angle *fOh* thus found by a straight *Os* meeting the horizontal trace used in point *s*; obtain *s'* in *xy*, and join *os*, *o's'*; these are the projections of the required bisecting line.

Problem XI. (See Fig. 16.)

To draw a straight line oh, o'h' *through a given point* pp' *to make a given angle θ with a given straight line* of, o'f'.

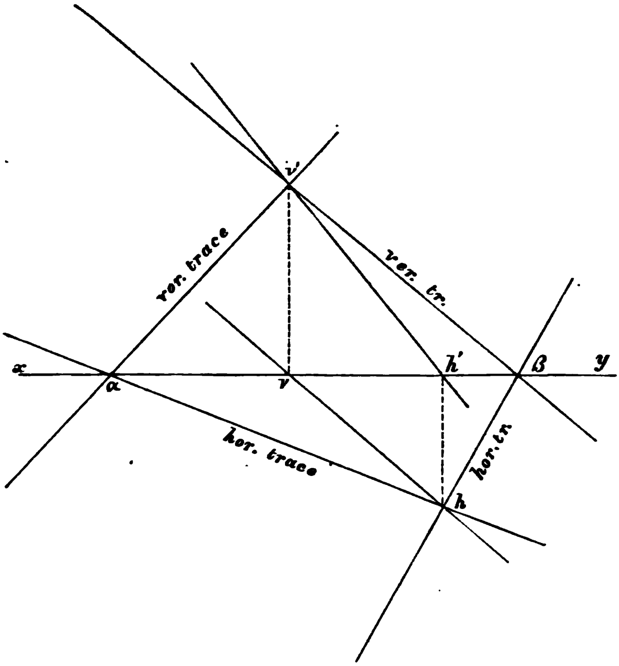
Determine the horizontal trace *fβ* of the plane of the given straight line *OF* and point *P* (Prob. 1, Cor. 1). 'Construct' this plane, and through *P* so 'constructed,' draw the line *OP* making the given angle *θ* with *OF* in the horizontal plane. Determine the projections *oo'* of point *O*, join *op*, *o'p'*, they are the projections of the required line.

Note: That the construction of this problem is made more simple by using a second plane of elevation perpendicular to that of the angle. Theorem III. (2).

PROBLEM XII.

To determine the projections of the intersection of two given planes hαv′ *and* hβv′.

Fig. 17.



The planes intersect in a straight line, which lying in both planes has its traces in those of the planes. Hence v', in which the vertical traces meet, is the vertical trace of the line of intersection, the *plan* v of this point is in xy, and is found by a perpendicular to it (Theorem I.). Similarly, *h* is the horizontal trace of the line required, and *h'*, its elevation, is found in *xy* by a perpendicular to that line.

Join *hv*, *h'v'*, these are the projections of the required line.

COR. 1. *To determine the intersection of three given planes.*

Determine the intersection *HV* as in the problem, of planes (1) and (2): then the intersection *AB* of planes (2) and (3). *AB* will meet *HV* in a point *P* which is the intersection required and is the vertex of the solid angle formed by the three given planes.

COR. 2. *To determine a straight line which shall pass through a given point* oo' *and meet two given straight lines* ab, a'b', *and* de, d'e'.

Determine the traces of the plane of the point *O* and the line *AB*; likewise those of the plane of point *O* and the straight line *DE*. Then the intersection of these two planes is the line required.

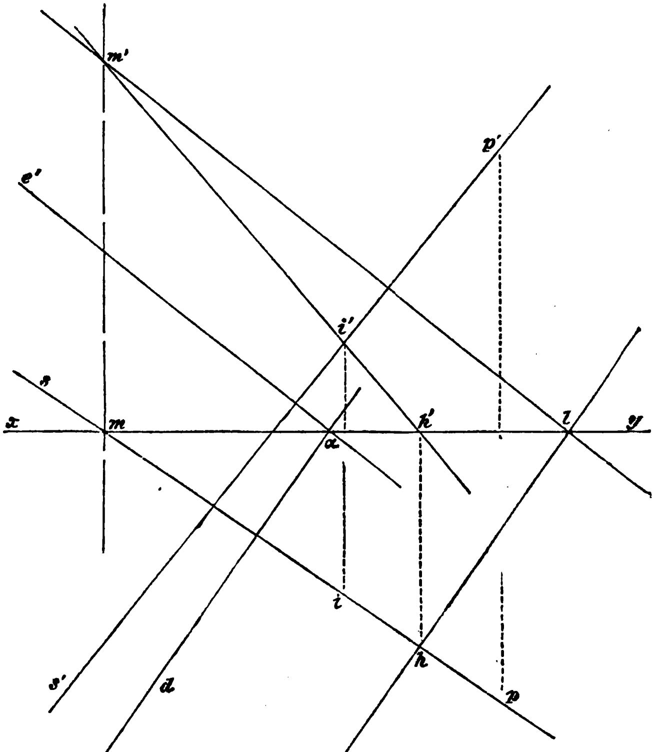
PROBLEM XIII.

To determine a straight line at right angles to a given plane qlm' *which shall pass through a given point* pp'. Fig. 18.

Since the projections of a perpendicular to a plane are perpendiculars to the 'traces' of the plane, Theorem V.; *ps* drawn through a point *p* at right angles to *ql*, and *p's'* at right angles to *lm'*, are the projections of the required line.

Conversely. To determine a plane at right angles to a given straight line *ps*, *p's'* which shall pass through a given point *ii'* in the same.

Fig. 18.



Draw the traces *da*, *e'a* of any plane *dae'*, by the problem, at right angles to the given line.

Determine the projections of any line lying in this plane and through the given point draw a parallel *ih*, *i'h'* to the line determined, and find its trace *hh'*.

Through *h* draw *ql* at right angles to *ps* meeting *xy* in *l*, and through *l* draw *lm'* at right angles to *p's'*: *qlm'* is the required plane.

COR. 1. *To determine the distance of* a *given point* pp' *from a given plane* qlm'.

Before reading this corollary study the following Problem No. 14, and by it determine point *ii'* in which the perpendicular *PS* to the plane intersects it.

Then by Problem 1 determine the length *IP* of the perpendicular of which *ip*, *i'p'* are the projections. *IP* is the required distance of the given point *P* from the plane.

COR. 2. *To determine two parallel planes at a given distance* (n) *inches from each other.*

Determine a straight line *BC*, and in it mark off the length *PQ*, points *P* and *Q* being (*n*) inches apart; the planes through these points at right angles to the line *BC* are those required.

PROBLEM XIV.

To determine the point of intersection of the given straight line ps, p's', *with the given plane.* Fig. 18.

Every plane which contains the given line meets the given plane in a straight line which passes through the required point of intersection of the given straight line and plane. Assume any such plane *pmm'*, of which *pm* and *mm'* are the traces.

Determine the line of intersection *hm*, *h'm'* of this assumed plane with the given one.

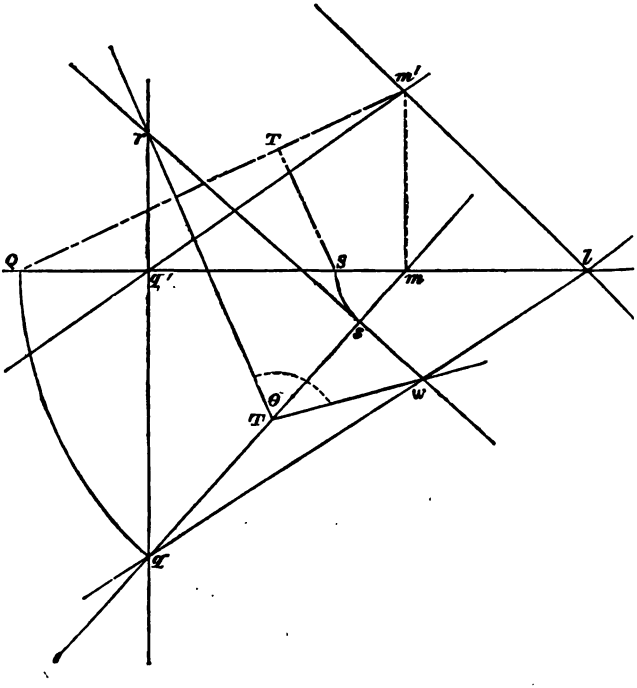
The projections *ii'* of the intersection of these two lines *HM* and *PS* are those of the required point.

Another mode of solving this problem is to make an elevation of the given line and plane on a plane at right angles to the latter: point *i'* in this elevation will be the intersection of the elevation of the line with the vertical trace of the plane: whence the plan *i* may be obtained.

PROBLEM XV.

To determine the angle which the two given planes qlm', qq'm' *make with each other,* i. e. *to determine their 'profile' or 'measuring angle.'*

Fig. 19.



This plane-angle is formed by two straight lines drawn one in each plane from a point in their common section *QM* at right angles to that line. (Euc. Bk. XI. def. 6.) The plane of these perpendiculars is therefore (Euc. Bk. XI. 4) at right angles to the line *QM*: hence if this plane be determined and 'constructed' about its horizontal trace, the required angle *wTr* is shown.

To do this, assume the horizontal trace *rw* of the plane of required angle, at right angles to *qm*, meeting it in *s*, and the horizontal traces of the given planes in *r* and *w*.

Fold the plane about *rw* bringing the point *T* in *QM* down into the horizontal plane in *qm*: join *Tr*, *Tw*.

The distance *Ts* is found by making it equal to *Ts* on the vertical plane (as shown), this line being the common section of the 'projecting plane' of the *plan* *qm* with the plane of the measuring angle.

Conversely. Given θ the profile or measuring angle of the inclination of two planes to each other, to determine the traces of the planes.

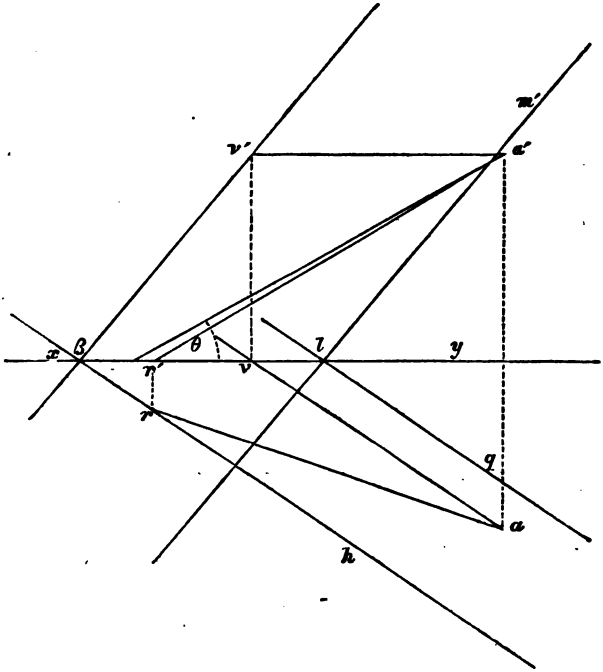
Draw the plane $qq'm'$ and determine the straight line qm, $q'm'$ in it.

Draw *rs* at right angles to *qm* and determine *sT* from *ST* as before and join *rT*: then make the angle $rTw = \theta$, and point *w* is determined, through which draw *ql* the horizontal trace of the second plane: lm' its vertical trace is drawn through the vertical trace of the line *QM* (Theorem VI.).

PROBLEM XVI.

Through a given point aa′ *to draw a plane parallel to a given plane* qlm′.

Fig. 20.



Since if two parallel planes be cut by a third plane, their sections with the third are parallel straight lines (Euc. XI. 16), therefore the traces of the given and required planes will be parallel straight lines.

If therefore the given plane be not parallel to *xy*, one point in either of the required traces being found both those lines can be obtained.

Through the given point *A* let a straight line be drawn parallel to either trace of the given plane; the trace of this line supplies the required point.

Therefore through *A* or *aa'* draw the straight line *A V* or *av*, *a'v'* parallel to *ql* which is a horizontal line; the trace *vv'* of this line gives the point *v'* in the vertical trace *βv'* of the required plane: and *βh* drawn parallel to *ql* through *β* gives the horizontal trace.

PROBLEM XVII.

Through a given point aa' *to draw a straight line which shall be parallel to a given plane* qlm' *and have a given inclination θ.* Fig. 20.

Obs. The angle *θ* cannot exceed the angle of inclination of the given plane.

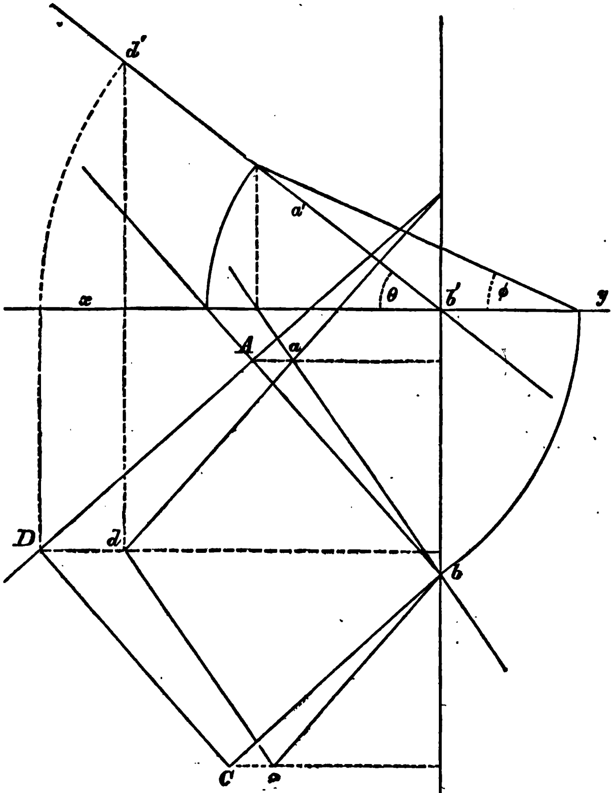
Through the given point *aa'* let a plane *hβv'* be drawn as in Problem 16, parallel to the given plane. And through point *A* draw the straight line *ar*, *a'r'* by Problem 8, to lie in the plane *hβv'*, and have an inclination of *θ* degrees.

Another mode of solving the problem is, to determine in the given plane a straight line having the required inclination, and to draw through the given point *A* a straight line parallel to the line so formed.

PROBLEM XVIII.

In a given plane bb′d′ *to determine a straight line which shall be perpendicular to a given straight line* ba, b′a′ *lying in that plane.*

Fig. 21.



'Construct' the given plane, revolving it about its horizontal trace *bb'* into the horizontal plane. Then *AB* will lie in the horizontal plane in the position *Ab.* Fig. 21.

The point *b* (or *B*) does not move during the rotation of the plane, being *in* the axis or trace *bb'* of the rotating plane. The perpendicular to *bb'* through point *a* is a locus of *A*, and the position of point *A in it* is found by taking the length *b'a'* along the vertical trace *b'd'* of the plane and setting it off from the line *bb'*. *A* and *b* joined gives the line *Ab* or *AB* 'constructed.'

Through point *A* draw the indefinite line *AD* perpendicular to *Ab*, and determine the plan *d* of any point *D* in this perpendicular. To do this:—Through *D* draw a line at right angles to *bb'* for one locus of *d* (Theorem X.). Measure the distance of point *D* from the line *bb'* and set this length from point *b'*, along the vertical trace of the plane, to determine *d'*. A perpendicular from *d'* to *xy* is a second locus of *d*. The intersection of these loci of *d* determine that point. Join *a* and *d*, then *ad, a'd'* are the projections of the required perpendicular.

Observe. In the same manner a straight line *AD* may be drawn in a given plane to make any given angle θ with a given straight line *AB* in that plane.

N.B. Theorems III. and IX. should be studied in connexion with this problem and figure.

PROBLEM XIX.

To determine the projections of a given plane figure, when the inclination of its plane is given and that of a straight line lying in the plane of the figure. Fig. 21.

1st. When the given line forms part of the perimeter of the figure.

Let the given figure be a square, its plane inclined θ degrees and one side *AB* of the figure inclined ϕ degrees.

Determine the plane *bb′d′*: in it place the line *ab*, *a′b′*. 'Construct' the plane as in Problem 18, and on *AB* describe the square *ABCD*: revolve the plane back into its former position and obtain *abcd*, the plan of the figure.

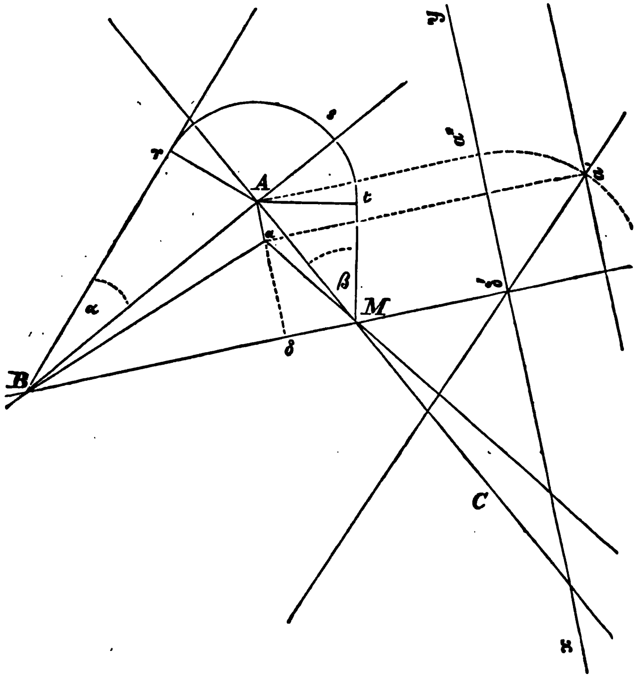
2ndly. When the given straight line is a diagonal of the given figure or its bisecting line: a tangent, a chord, or a diameter of a given ellipse or circle.

Determine the plane *bb′d′* of the figure and the projection of the given line in it. 'Construct' the plane, and the line as lying in it; draw the given figure in the horizontal plane having the required position with regard to the 'constructed' line: and from thence proceed as before to obtain the projections of the entire figure when turned back into the inclined position.

PROBLEM XX.

Given two straight lines AB, AC *making an angle* BAC *of θ degrees with each other; to determine the projections of the lines when* AB *is inclined at an angle of a degrees and* AC *inclined β.*

Fig. 22.



Observe the conditions of the problem are possible if $a + \beta + \theta$ exceed 180°:

if $\alpha + \beta + \theta$ equal 180°

the plans of AB and AC will be in one straight line, *i.e.* the plane of the angle BAC will be a perpendicular one.

The case in which $\alpha + \beta + \theta$ is less than 180° will be that here considered.

Draw the angle BAC on the horizontal plane. It will be necessary to revolve the plane of the angle BAC into the inclined position it will occupy when its containing lines have the required inclinations. For this purpose a horizontal line or trace BM of the plane must be discovered, about which the plane is to revolve. Assume a point B in AB and

make the angle $ABr = \alpha^\circ$ and
... ... $ArB = 90^\circ$.

Then Br will be the length of the plan of BA when BA is in the required position.

Now for the same height $Ar \, (= At)$, by the aid of a tangent tM to the arc rst determine

the angle $AtM = \beta^\circ$ and
... ... $AtM = 90^\circ$: and point M is found.

The straight line BM is the required horizontal trace of the given angle. A plane of elevation having its xy at right angles to this line will be always perpendicular to the plane of that angle; and the elevation of the angle upon it will be in the vertical trace, Theorem III. 2.

The elevation of the path of point A is the circular arc $a''a'$, in which a' is determined by the parallel to xy at the distance Ar.

From a' the plan a is found in $A\delta$ (Theorem I.).

Then BaM is the required plan, and $\delta' a'$ the elevation and vertical trace of the plane of the given angle. $a''\delta' a'$ is the angle of the inclination of the plane of the lines.

COR. *To determine three straight lines meeting in a point, each line perpendicular to the other two.*

Proceed, as in this problem, to obtain the projections of the two lines AB and AC; and through point aa', by Problem 13, determine the projections ae, ae' of the third line, at right angles to the plane $M\delta a'$. AB, AC, AE are the required lines.

PROBLEM XXI.

To determine the projection of a given plane figure when the heights of three points in the plane of the figure are given.

Limits. When the given heights are equal, the projection of the figure is an equal and similar figure to the given one.

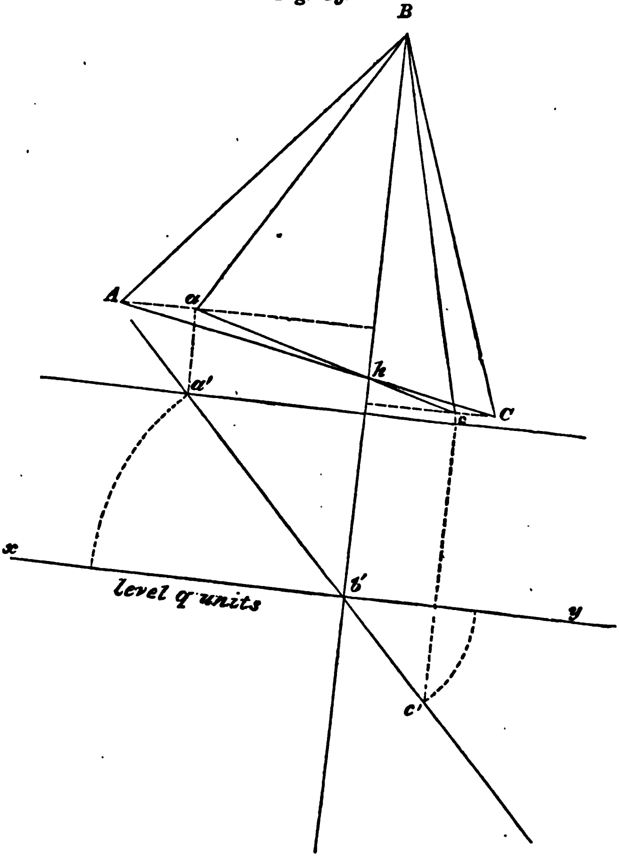
When the given heights are unequal, the problem is a possible one only when the difference between the heights of any two points does not exceed the distance between these points.

If this difference is equal to that distance the plane of the figure is vertical; and the projection required will be a straight line.

Let the three given points A, B and C be at the angles of an equilateral triangle of m inches side, and the heights of A, B and C be p, q and r inches respectively: to determine the plan abc of the triangle. Fig. 23.

(1) Draw ABC in the horizontal plane.

Fig. 23.



(2) Determine point *h* in *AC* which will have the same height as *B*, viz. *q* inches. Then *hB* is a horizontal line when the triangle is in its inclined position.

(3) By the aid of an elevation at right angles to *hB* as shown, the plan *abc* is determined.

PROBLEM XXII.

To determine two parallel planes which shall contain the given straight lines AB *or* ab, a'b' *and* CD *or* cd, c'd'.

1st. If the given lines *AB* and *CD* are parallel.

Determine the horizontal traces of the lines; parallel lines through these will be the horizontal traces of the parallel planes required, whence the vertical traces can be obtained.

2ndly. If the given lines *AB* and *CD* be neither parallel nor in the same plane.

Through any point *A* in *AB* draw a straight line *AQ* parallel to *CD*.

Determine the horizontal traces of the lines *AB* and *AQ*; the straight line drawn through these points is the horizontal trace of one of the required planes.

Determine the horizontal trace of *CD*; through this point a parallel line to the trace just found is the horizontal trace of the second plane required: and the vertical traces of the two planes can be obtained.

3rdly. If the given lines be not parallel but in the same plane, the problem is impossible.

PROBLEM XXIII.

To determine a straight line at right angles to each of two given straight lines AB *and* CD.

1st. If the given lines be parallel.

Any straight line drawn in the plane of the parallels perpendicular to one of the lines is also perpendicular to the other.

2ndly. When the given lines are neither parallel nor in the same place.

From any point *P* in *AB* draw a line *PQ* parallel to *CD*: then the plane of *AB* and *PQ* is parallel to *CD*.

From any point *C* in *CD* draw *CS* perpendicular to the plane of *APQ*, meeting it in *S*. A parallel to *PQ* or *CD* through point *S* meets *AB* in *H*. Then *HT*, parallel to *CS* from *H*, meets *CD* in point *T* and is the required perpendicular to the given lines.

PROBLEM XXIV.

To determine the angle of inclination θ *of a given straight line* ab, a'b' *to a given plane* qlm'. Fig. 24.

In this problem it is convenient to determine the complement of the angle required.

This complementary angle is the angle between the given straight line and a line meeting it in point *A* and perpendicular to the given plane.

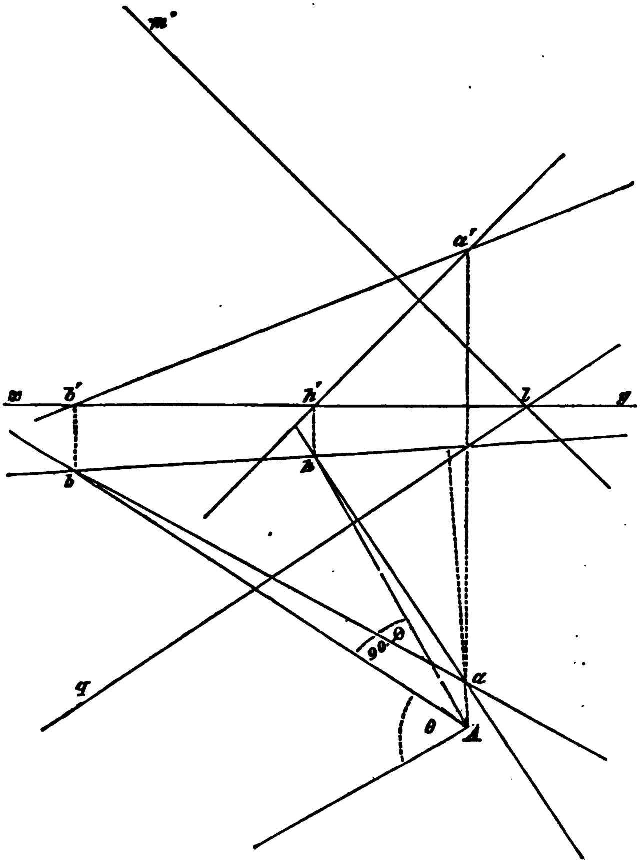
Determine this perpendicular *ah*, *a'h'*, by Problem XIII., and find its horizontal trace *ha'*; likewise *bb'* that of the given line; the line *hb* through these points is the horizontal trace of the plane of the lines. 'Construct' their plane about *hb* bringing *A* into the horizontal plane. *Ah*, *Ab*, when drawn, give *bAh* the complementary angle or $90 - \theta$, and θ is the angle sought.

Conversely:

To determine a straight line through a given point aa', *to make a given angle* θ *with a given plane* qlm'.

Through *aa'* draw *ah*, *a'h'* at right angles to the given plane, and determine its horizontal trace *hh'*. Through

Fig. 24.



point *h* draw in any direction the line *hb* as the horizontal trace of the plane of the perpendicular and the required line. Fold this plane about *hb* into the horizontal one and through point *A* thus 'constructed' let *Ab* making an angle

$90 - \theta$ with Ah meet the trace hb in point b. Then b' being found in xy and joined to a', the required line ab, $a'b$ is determined.

Observe. An infinite number of such lines ab, $a'b'$ can be determined to satisfy the given conditions; and their 'locus' is the right-conical surface having its circular base in the given plane and its generatrix inclined θ degrees to the same.

PROBLEM XXV.

To determine two planes which shall be perpendicular to each other and inclined at angles of θ and ϕ degrees respectively to the horizontal plane.

It will be obvious that the angle of inclination of the second plane cannot be less than the complement of that of the first; or ϕ not less than $90 - \theta$.

Therefore

if ϕ is greater than $90 - \theta$, there are two solutions:

...ϕ is equal to $90 - \theta$, there is one solution;

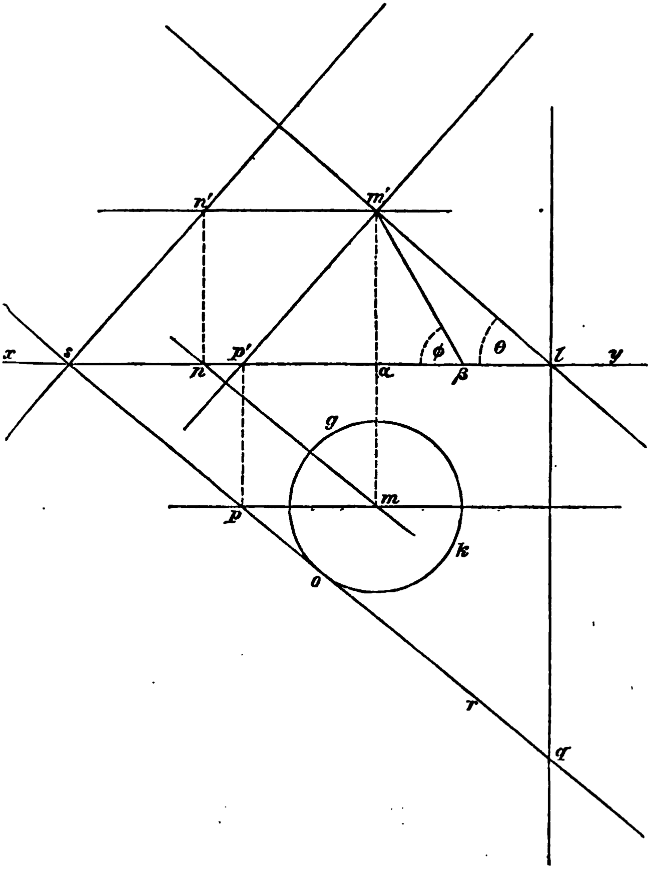
...ϕ is less than $90 - \theta$, there is no solution.

This will appear by what follows.

Draw the plane qlm' inclined θ degrees; also the straight line pm, $p'm'$ at right angles to it (Problem 13), and find the horizontal trace pp' of PM.

Then every plane which contains that line is at right angles to the plane θ (Eucl. XI. 18). Wherefore if such a plane revolve about PM, as an axis, until it makes an angle ϕ with the horizontal plane, it is the plane required.

Fig. 25.



Let it therefore be made to touch a right cone, vertex in any point *mm'* in *PM* and generatrix *m'β* making an angle of ϕ degrees with its base *gko*. The horizontal trace of this plane must touch *gko*; (Theorem VII.).

And since the plane is to contain *PM*, the horizontal trace *rp* of the plane must pass through *p*, (Theorem VI.).

Through the point *mm′* draw the horizontal line *mn*, *m′n′* parallel to the trace *rp*, and determine its vertical trace *nn′*.

Through *n′* and *s* draw the vertical trace of the plane.

Observe that the plane *rsn′* by containing *PM* is perpendicular to plane θ; and, by touching the cone described, has the required inclination ϕ; it thus satisfies the conditions of the problem.

Note also that the inclination of *PM* is $90 - \theta$; since if a straight line and an inclined plane be perpendicular to each other their inclinations are complementary.

Cor. *To determine three planes each perpendicular to the other two.*

For the first and second, determine planes *qlm′* and *rsn′* as in the above problem.

Find their intersection *mq*, *m′q′* by Problem 12.

The plane which is perpendicular to *MQ* is perpendicular to the planes which meet in this line (Eucl. XI. 19). Therefore such plane, determined by Problem 13, is the 3rd plane required.

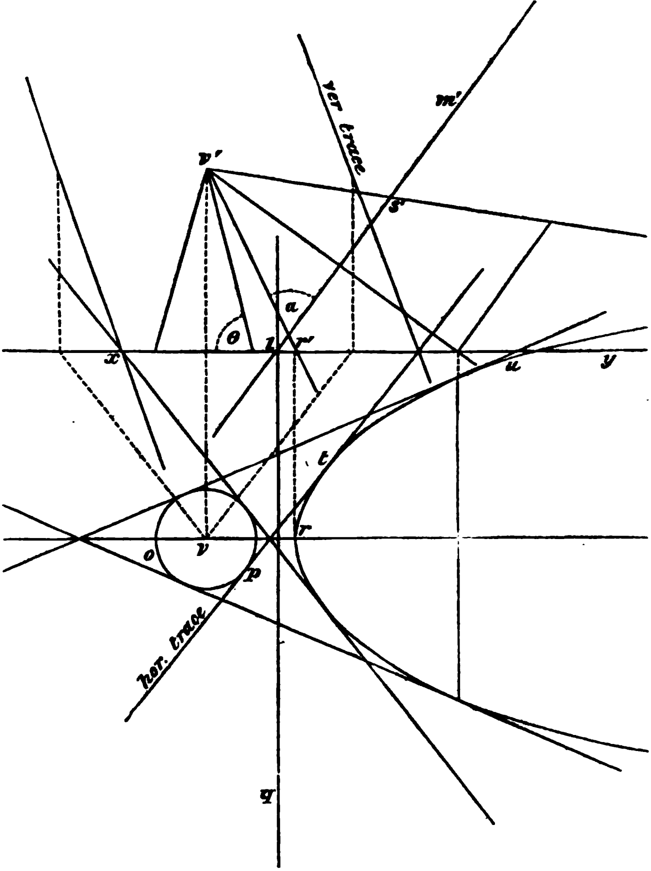
PROBLEM XXVI.

To determine a plane inclined at an angle of θ degrees and making a given angle α with a given inclined plane.

Let *qlm′* be the given inclined plane.

To fulfil both conditions, the required plane must touch two right cones having a common vertex in any point *vv′*

Fig. 26.



with circular bases op and $b's'$ in the horizontal and inclined planes respectively[*]; the generatrix of the former making an angle θ with its base, and that of the latter an angle of a degrees with the inclined plane.

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[*] b' is the point of intersection of the lines $v'r'$ and lm', and is not lettered in the fig.: also the plan of the circular base $b's'$ is not shown in the fig.

The horizontal trace *pt* of the common tangent plane to the cones is (Theorem VII.) a tangent to the horizontal traces of the two conic surfaces, i.e. the trace *pt* touches the circle *op* and the ellipse *rtu*. Since four such lines can be drawn, as appears by the figure, it is obvious that in the case here indicated four planes can be determined which satisfy the given conditions of the problem.

Should the circle *op* fall within the ellipse, without touching it, the given conditions are impossible.

PROBLEM XXVII.

Given three lines ag, ak, ar *meeting in a point* a*; as the plans of the straight lines* AG, AK, AR, *each of which is at right angles to the plane of the other two; to determine an elevation of the lines.*

Since the plane of lines *AK*, *AR* is perpendicular to the line *AG* (Eucl. XI. 4), the horizontal trace, *cd*, of the plane of *AK*, *AR* is perpendicular to *ag*, the plan of *AG*; (Theorem V.).

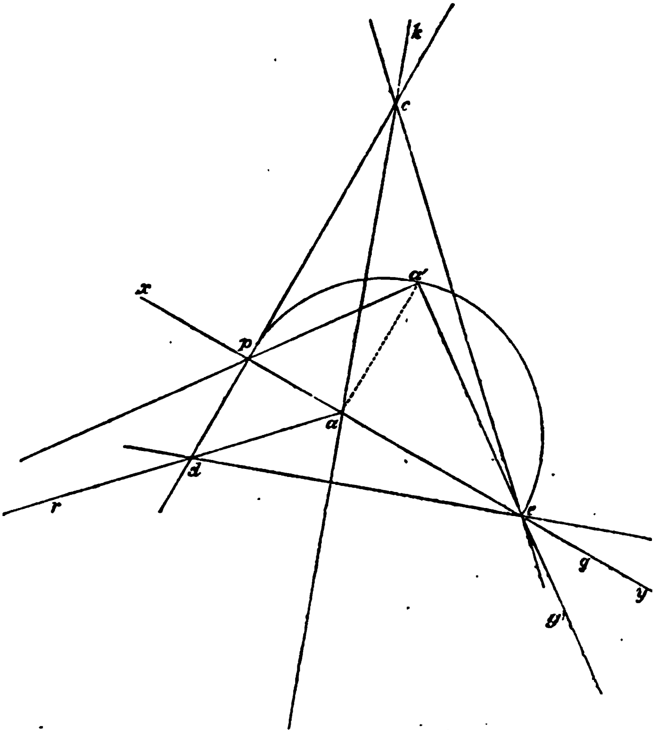
Therefore draw *cd* at right angles to *ga* produced, and meeting *ak*, *ar* in points *c* and *d*.

ce drawn through *c* at right angles to *ar*, and *de* drawn through *d* at right angles to *ak*, will meet *ag* in *e* and be the horizontal traces of the other two planes; (Theorem V.).

Take the projecting plane of the plan of *AG* for a plane of elevation, and let it revolve about the line *ag* as an *xy*.

This plane cuts the plane of *AK* and *AR* in *AP* or *a'p*, which is therefore perpendicular to *AG* or *a'g'* in the plane of elevation; (Eucl. XI. def. 3).

Fig. 27.



Therefore on *ep* describe the semicircle *ea'p*, through *a* draw *aa'* perpendicular to *xy*, and *ea'*, *a'p* are the required elevations of the three lines; (Theorem III. 2).

PROBLEM XXVIII.

Given two points B *and* C *at unequal distances from a given plane, to determine a point* Q *in the plane at which the straight lines* QB *and* QC *shall make equal angles with the plane.*

From either of the points, as B, draw the straight line BR at right angles to the given plane (Problem 13), meeting it in S, determined by Problem 14.

Make $SR = SB$, point R being on the opposite side of the plane to B; join R, C, cutting the given plane in point Q: QB, QC are the required lines.

Exercises on Chapter III.

1. Determine the projections of four points, each 2 inches from the planes of projection, viz. one in each dihedral angle.

2. Determine four points each in a plane of projection and 3 inches from xy; no two of them to be on the same side of that line.

Supposing the loci of these points (Theorem I.) to be 2 inches apart, determine the distance between any two points.

3. A point is 2·5 inches from the planes of projection, draw two lines through it inclined 50°; their plans making angles of 35° with xy. Problem 1, converse (2).

Determine the traces of the lines: also the angle which the lines make with each other. Problem 10.

4. Through a point 2 inches from each plane of projection draw lines inclined 30°, making angles of 40° with the vertical plane. Problem 3 and Theorem IV.

5. A straight line in the horizontal plane makes an angle of 45° with *xy* and is a trace of planes which are inclined 50°; find their vertical traces. Problem 3, "tangent planes to cones," Chap. VIII.

6. A regular hexagon, side 2 inches, has its diameter and adjacent side inclined 30° and 45° respectively: draw its plan and determine the inclination of its plane. Problem 20.

7. Draw two parallel planes 2 inches apart and inclined 60°. Problems 1 and 13, and Problem 25, *note.*

8. A plane is inclined 40°, and is at right angles to another which is inclined 70°: draw the traces of the planes. Problem 25.

9. The horizontal traces of two planes, inclined 45° and 30° respectively, make an angle of 60°: determine a third plane at right angles to the given ones. Problems 12 and 13.

10. Two planes are inclined 60° and 70° respectively and make an angle of 50° with each other: determine their vertical traces on a plane not perpendicular to them. Problem 26.

11. A regular pentagon, side 2·5 inches, has three of its angular points respectively 1, 2, and 3 inches high. Draw its plan. Problem 21.

12. A square, side 3 inches, lies in a plane inclined 45°, one side of the figure makes an angle of 40° with the horizontal trace of its plane. Draw its plan. Problem 18.

13. A plane is inclined 80° and makes an angle of 60° with a plane which is inclined 50°. Draw the traces of the planes. Problem 26.

14. The two traces of a plane make an angle of 50° with each other and make equal angles with *xy*: determine these latter angles and the inclination of the plane.

15. Three straight lines are perpendicular to one another and two of them are inclined 25° and 40°: determine the inclination of the third line. Problems 20 and 13.

16. The horizontal and vertical traces of a plane make angles of 30° and 55° respectively with *xy*; a straight line parallel to *xy* is 2 inches from the vertical and 3 inches from the horizontal plane: find the intersection of the given line and plane. Problem 14.

17. Two planes contain a right angle; one of them is inclined 60° and their intersection is inclined 50°. Draw the traces of the planes and find the inclination of the second. Problems 8 and 13.

18. The hypothenuse of a right-angled triangle is 4 inches long, and horizontal; the plans of the sides make an angle of 115° with each other: determine the lengths and inclinations of the sides. Can more than one set of answers be given? Eucl. III. 33, and Problem 20.

19. A straight line inclined 40° lies in a plane inclined 60°; determine a plane containing the given line and perpendicular to the given plane. Problems 8 and 13.

20. Through a point 3 inches high draw three planes, inclinations 35°, 45°, and 60° respectively; forming a tri-

hedral angle at the given point. Show the real magnitude of the angle of each face of the solid angle. Problems 12 and 10.

21. The plans of two lines contain an angle of 110°. The lines themselves are at right angles, and one is inclined 27°. Find the inclination of the other. *Science Exam. Hon.* 1872.

(1) Draw plans of the lines and make an elevation of one, assumed as that inclined 27°, on a vertical plane taken parallel to it.

(2) Determine a plane perpendicular to this line and containing the other. The point in which the horizontal trace of this plane meets the plan of the second line is the horizontal trace of the line whose angle of inclination is required.

(3) Determine this angle by Chap. III. Prob. I.

22. Two lines are inclined at 30° and 40°. They are 2 inches apart where they are nearest together, and this line of 2 inches is inclined 28°. Show plan and elevation. *Science Exam. Hon.* 1873.

(1) Determine a line inclined 28°, and mark off a segment of this line 2 inches in length. Chap. III. Prob. I.

(2) Draw two parallel planes at right angles to line of 28° passing through the extremities of the segment of 2 inches.

(3) From the point where line of 28° meets one plane draw a line lying in that plane and inclined at 40°, and from the point where the former line meets the parallel plane draw another line lying in that plane and inclined 30°.

The two lines so drawn are those required.

Note. Four pairs of lines fulfilling the conditions are possible.

23. Given a plane by its traces, and two lines, not passing through the same point, by their projections. The lines to be neither in the given plane nor parallel to it. Find the projections of a line which shall meet the two given lines, be parallel to the given plane, and at a given distance from it.

Determine a plane parallel to the given plane at a distance from it equal to the given distance of the parallel line from the same plane. See Chapter III. Problem 13, Cor. 2.

Find the points of intersection of the two given lines with this plane. See Chapter III. Prob. 14. The line joining the two points thus found is the line required.

24. Two horizontal lines AB, AC, contain an angle of 56°, a plane inclined at 30° contains AB, another inclined at 65° contains AC. Draw two lines passing through A, each inclined at 20° and lying one in each plane. Determine the angle between these two lines. *Science Exam. Hon.* 1872. Chapter III. Problems 4, 8 and 10.

25. A line TV 3·3 inches long has its extremities T and V in the horizontal and vertical planes respectively, it is inclined to the horizontal plane at 40°, and has its trace T at ·75 inch from the ground line. Draw its plan and elevation. *Science Exam.* 1868.

26. An indefinite line is inclined to the horizontal plane at 40° and makes an angle of 30° with the vertical

plane of projection, the distance between its traces being 3·5 inches. *Science Exam. Hon.* 1869.

27. *AB* is a line parallel to the horizontal plane, *AC* is a line parallel to the vertical plane. The angles *bac* between the plans and *b′a′c′* between the elevations are 120°. Find the real angle between the lines. *Science Exam.* 1871.

28. Determine a line inclined at 33° lying in a plane inclined at 50°. This is the orthographic projection on that plane of a line which makes an angle of 40° with it. Determine the projections of this latter line on the coordinate planes and its inclination. *Science Exam. Hon.* 1871.

IV.

SOLIDS WITH THE INCLINATIONS OF THE PLANE OF ONE FACE AND OF ONE EDGE IN THAT FACE GIVEN.

BEFORE commencing this chapter the student should refer to Chapter III. Problems 18 and 19.

1. *Plan and elevation of a pyramid,* 4 *inches high, with square base of* 2·5 *inches side, when its base is inclined* 50°, *and no side horizontal.*

(1) Determine by its traces a plane 50° at right angles to the vertical plane; i.e. draw its horizontal trace at right angles to, and its vertical trace at 50° with, the ground line.

(2) "Construct," that is, fold this plane on its horizontal trace into the horizontal plane, and draw the square as then in the horizontal plane; i.e. simply draw the square, taking care that no side is parallel to xy.

(3) Lift the plane back to the required position and determine required plan and elevation of the square base. Remember that this plane being at right angles to the vertical plane will have all points in it shown in this elevation *in* the vertical trace (Theorem III. 2). The square base will therefore have its *elevation* wholly in the vertical trace.

To draw this elevation, take the distances from the corners of the square to the horizontal trace when the plane is folded down, and set them along the vertical trace from its junction with *xy*. This will give these points in *elevation.* Their *plans* will lie in the perpendiculars to *xy* from their *elevations* (Theorem I.) and also in the perpendiculars to the horizontal trace from the points as "constructed" (Theorem X.). Thus let the point of which *a′* is the elevation when folded down or 'constructed' be marked *A*. A perpendicular from *a′* to *xy* contains the plan *a*, as also does the perpendicular from *A* to the horizontal trace. The plan *a* therefore lies in the intersection of these perpendiculars; in other words, these perpendiculars are loci of the point *a*, and their intersection gives the point itself.

(4) The axis will be drawn from the centre of the base in plan and elevation at right angles to the traces, and being parallel to the vertical plane will be set in true length *in elevation.* It only remains to join the proper points and the projections of the solid are completed.

2. *Plan and elevation of a cube of* 2·5 *inches edge, when one face is inclined* 60°, *and no side horizontal.*

3. *Plan and elevation of a pyramid,* 4 *inches high, with regular hexagonal base of* 2·5 *inches side inclined* 70°, *no side horizontal, and its lowest corner* 1 *inch above the ground.*

4. *Plan and elevation of the pyramid in the last problem when the base is inclined* 70° *and one side of the base* 30°.

In this problem there is given not only the inclination of the plane of the base, but also of one edge in that base.

LIMITS. The problem is therefore to draw a line of given inclination in a plane of given inclination, and this is always possible if *the inclination of the line is not greater than that of the plane.*

(1) Determine by its traces the required plane inclined 70° and at right angles to the vertical plane. Assume any point *A* in the plane; its elevation *a*′ is by the position of the plane in the vertical trace; its plan *a* on a perpendicular to *xy* from *a*′, and for convenience it, i.e. the plan *a*, may be assumed at the point where this perpendicular meets *xy*; and from *a*′ draw *a*′*m* to meet the ground line at the given angle 30°. Rotate the triangle thus formed on the vertical line *a*′*a*, the projector of the point *A*, until the point *m* meets the horizontal trace. As *A* and *m* are now both in the given plane, the line joining *a* and *m* will be the plan of *Am* and will fulfil the conditions by being the plan of a line wholly in plane 70° and itself inclined 30°.

(2) 'Construct' plane 70° on its horizontal trace into the horizontal plane. On the line *Am*, thus brought into the horizontal plane, describe the required hexagon, and complete plan and elevation of the whole as in former problems.

5. *Plan and elevation of a cube of* 2·5 *inches edge when the plane of one face is inclined* 65° *and one diagonal of that face* 25°.

6. *Plan and elevation of a tetrahedron of* 3 *inches edge with one face* 70° *and one edge in that face* 40°. *Determine also the inclinations of the other edges of the solid.*

7. *Plan and elevation of hexagonal right prism when the plane of one of its faces* ABCD *is inclined* 50° *and the edge* AB *of the base inclined* 35°.

(1) Determine the plan of rectangular face *ABCD* as in preceding problems.

(2) It will next be necessary to determine the plane of the base, which, by the definition of the solid, is perpendicular to the plane of the face. The line *AB* already determined is in the base, being common to both planes, and its horizontal trace will therefore be a point in the horizontal trace of the required plane of base. And as the projections of a perpendicular to a plane are perpendicular to its traces, the horizontal trace of the plane of the base may be at once drawn through this point at right angles to the long edges of the prism, that is, in this case to *bc* or *ad*.

(3) Take a ground line at right angles to the horizontal trace of plane of base, and set up the height of *A* or *B* (both known points) in the new elevation. By the position of the new plane of elevation, the vertical trace will contain the elevations of both *A* and *B*, and therefore can at once be drawn.

Fold the plane of the base about its horizontal trace to bring *AB* down into the horizontal plane. On the *AB* thus constructed describe hexagon and complete as in former problems.

8. *The plane containing one edge of a tetrahedron and bisecting another is inclined at* 50°, *and the former edge is inclined at* 30°. *Draw plan and elevation of the solid, the edge being* 3 *inches.* Science Exam. Hon. 1870.

(1) Determine plane 50° containing line inclined 30°, and "construct" it.

(2) Measure off on "constructed" line 3 inches, and on this as a base draw an isosceles triangle having its equal

sides equal to the altitude of an equilateral triangle of 3 inches side. The figure thus drawn is the true form of the section of the tetrahedron made by the given plane.

(3) Reconstruct plane, and from vertex of triangle draw a perpendicular to the plane and produce it on both sides to equal distances of 1·5 in. On joining the extremities of this perpendicular with the extremities of the line of 3 inches measured off on line of 30°, we obtain the projections of tetrahedron required.

V.

SOLIDS WITH THE INCLINATIONS OF TWO ADJACENT EDGES GIVEN.

LIMITS. It must be remembered that *the sum of these inclinations with the angle contained by the two edges must not exceed* 180°.

For these problems refer to Chapter III., Problem 20.

1. *Plan and elevation of a cube when the inclinations of two adjacent edges as* BA, BC, 20° *and* 48° *are given.*

(1) Draw the square $ABCD$ to represent the face '*constructed*' into the horizontal plane. Then to find a horizontal in the plane of this face. At any point E in BA, or BA produced, make the angle BEF 20°. Then if a perpendicular be drawn from B to EF, it will be the length of the line expressing the height to which B, when in required position, will be raised above horizontal plane containing point E. From B as a centre with this perpendicular for radius, describe a circle. The tangent to this circle to make an angle of 48° (i.e. the inclination of the other edge) with BC will give a point H where it meets BC in the *horizontal* required. E being another point in it, the straight line through H and E will be the horizontal sought in plane of face $ABCD$.

(2) Assume a plane of elevation at right angles to this inclined plane of face *ABCD*. The height to which point *B* is to be raised being known, a line of level that distance above the *xy* taken will contain the elevation of *B*. The rest of the work is similar to that in preceding problems.

NOTE. To understand the principles of this construction, the student had better cut a card-board model, by which to illustrate the folding up and down of the plane of the angles and of the lines.

2. *Plan and elevation of a tetrahedron when two of the adjacent edges in one face are inclined* 28° *and* 62°.

3. *Plan and elevation of hexagonal right prism when the two edges* AB, AC *in one face are inclined respectively* 25° *and* 40°.

Determine the plan of the rectangular face *ABCD* as in Problem 1, and refer to explanations of Problem 7, Chapter IV., to find the plane of base and to complete the projections of the solid.

VI.

SOLIDS WITH THE INCLINATIONS OF TWO ADJACENT FACES GIVEN.

For these problems refer to Chapter III. Problem 25.

Limits. Case 1. If the given planes are at right angles to each other, their inclination being θ° and ϕ°, ϕ° must be not less than $90^\circ - \theta^\circ$.

Case 2. If the given planes are not at right angles to each other, refer to Chapter III. Problem 26.

1. *Plan and elevation of a cube of* 3 *inches edge, when the inclinations,* 45° *and* 60°, *of the planes of two adjacent faces are given.*

(1) Determine the traces of plane 45° assumed at right angles to the vertical plane.

(2) Determine the projections of a perpendicular to plane 45°, in order to determine plane 60°, since every plane containing this perpendicular is at right angles to the former plane.

(3) Determine *horizontal trace* of plane 60°. To do this, make the point at which the perpendicular pierces plane 45° the vertex of a right cone, generatrix 60°, base in horizontal plane. Then because every plane touching

this cone is inclined 60°, a tangent to the circular base of the cone will be the horizontal trace of such a plane, and if it be drawn as well through the horizontal trace of the perpendicular to plane 45°, it will be the horizontal trace required, i.e. of a plane perpendicular to plane 45° and inclined itself 60°.

(4) Determine the *plan* of the intersection of the two planes. One point in this plan will be at the intersection of the horizontal traces of the two planes, and a second point will be the plan of the vertex of the cone. The line drawn through these two points will be the plan of the intersection required. 'Construct' this line into the horizontal plane, draw the square face of the cube on it so 'constructed,' and determine plan of the face therefrom. Complete plan and elevation of the cube as in former problems.

2. *A prism 3 inches long has an equilateral triangle of 2·3 inches side for its base; draw its plan and an elevation of it when the planes of its base and one face are equally inclined at 60° to the horizontal plane.* Science Exam. Hon. 1869.

3. *Plan and elevation of a prism with base a regular hexagon. Plane of one face inclined 48°. Base 68°.*

(1) Determine the plan of the rectangular face of prism as for the face of the cube first determined in the last problem.

(2) On a vertical plane at right angles to plane 68°, i.e. with its ground line taken at right angles to the horizontal trace of plane 68°, determine the vertical trace of plane 68°, and 'construct' this plane into the horizontal plane. On *AB*, the edge common to plane of face and plane of base so 'constructed,' describe the

regular hexagon. Determine plan therefrom and complete problem as in former cases.

4. *Plan and elevation of a tetrahedron. Planes of adjacent faces θ and ϕ.*

For example, let $\theta = 40°$ and $\phi = 70°$.

(1) Determine the angle α between the two adjacent faces of the solid.

(2) Draw the traces of plane θ, taking the plane at right angles to the vertical plane of elevation.

(3) Determine a second plane inclined ϕ and making an angle of α with plane θ, Chapter III. Problem 26. The intersection of these two planes θ and ϕ is the indefinite edge of the solid. 'Construct' plane θ or ϕ, and on the intersection so 'constructed' mark off the length of the given edge of the tetrahedron, describe the face upon it, and proceed as in former problems.

5. *Plan and elevation of a tetrahedron with one face vertical and an adjacent face inclined* 50°.

Determine the angle θ between adjacent faces.

Determine a plane inclined 50° to the horizontal plane and θ to the vertical plane by Chapter III. Problem 7.

On the vertical trace place $a'b'$ the real length of an edge of the tetrahedron, and on it describe the equilateral triangle $a'b'c'$. Find the centre v'. $a'b'c'v'$ is the required elevation.

In plan the vertical face $a'b'c'$ will be a straight line abc parallel to xy, and the plan b of the vertex will be a point in the projector drawn from b' perpendicular to xy at a distance in front of abc equal in length to the axis of the tetrahedron. Lines joining b with the points abc of the vertical face complete the plan of the solid.

VII.

SECTIONS BY OBLIQUE PLANES.

1. *To find the section of a prism standing on one of its bases when cut by an oblique plane given by its traces.*

Draw the plan and elevation of prism and the plane of section.

Assume auxiliary vertical planes with their horizontal traces parallel to the horizontal trace of the given cutting plane, and containing the edges of the prism. The points in which these edges are cut by the plane of section will be in the intersection of this plane with the assumed vertical planes. The heights at which they are cut will be shown by the heights at which the vertical traces of the sectional plane and assumed planes cross. These heights can readily be transferred to the proper edges in elevation, and the section required completed by joining adjacent edges.

Another method.

Determine the intersection of the planes of the faces with the plane of section. The elevations of these intersections will cut the elevations of the edges of the prism in the required points of section.

mark on them from the vertical trace the actual distances of the points *P*, &c. from that trace, i.e. the hypothenuses of the right-angled triangles of which perpendiculars from p', &c. on vertical trace, and from p, &c. on xy, are the sides. Join the consecutive points for the section.

For the rest of the frustum, i.e. for the points *A*, *B*, &c. of the base,

(1) Determine perpendiculars from the points of the base *A*, *B*, *C*, &c. to the plane of section.

(2) Find intersection of these perpendiculars with that plane, as oo', the point where the perpendicular through *A* intersects the plane.

(3) Then proceed with these points of intersection as with the points of the section of the solid.

To save work, when one point as o'' in the required projection of the base is found, join o'', p'', and draw parallels to this line through the other points q'', r'', &c. Perpendiculars to vertical trace through the elevations b', c', &c. will intersect these parallels in the points b'', c'', &c. required, which may be joined consecutively to complete the projection sought.

2. *Take a cylinder resting on its base and cut it by an oblique plan. Determine the true form of the section and the development of the frustum of the solid.*

Divide the circumference into any number of parts, and work as if the points so obtained were the plans of the edges of a prism. The section will be an ellipse.

Converse of this Problem.

Given ab *the major axis of the ellipse forming the section of a right cylinder by the horizontal plane when the axis of the solid is inclined* θ *degrees; to determine base and plan of frustum of solid.*

To determine the minor axis of the ellipse, which it may be observed is equal to the diameter of the required base.

The vertical plane through *ab* contains the axis of the cylinder, and is at right angles to its base. Revolve this plane into the horizontal. Bisect *ab* in *o*, and draw *op'* making the angle *bop'* equal θ degrees. Through *p'* draw *l'p'm'* at right angles to *op'*, and meeting parallels to that line through *a* and *b* in *l'* and *m'*. Then *l'm'* gives the length of the required minor axis *cd*. Draw the curve.

Through *p'* a perpendicular to *ab* produced meets that line in *p* which is the centre of the plan of the base of the cylinder, which plan is likewise an ellipse having its major axis *nq* equal and parallel to the minor of the former; its own minor *lm* being determined by perpendicular on *ab* drawn from *l'* and *m'*. Tangents to these two curves complete the plan of the frustum of the cylinder.

3. *Given an ellipse as the horizontal trace of a cylinder. Find its plan, and an elevation on a vertical plane making an angle of* 30° *with plan of axis.*

(1) Describe a semicircle on the major axis *ab* and from one extremity as *a* set off the length of the minor axis *cd* as a chord of the semicircle to *m*. The line drawn from *o*, the centre of *ab*, at right angles to *am*, is an auxiliary eleva-

tion of the axis of the required cylinder on a vertical plane passing through *ab*.

Parallels to the elevation of this axis from *a* and *b* complete the auxiliary elevation of the indefinite cylinder. Parallels from *c* and *d* to *ab* produced complete the plan of the indefinite cylinder.

(2) Draw *xy* making an angle of 30° with plan of axis and on the vertical plane passing through *xy* make a new elevation. To do this, assume any point *P* in the auxiliary elevation of the axis and find its plan *p*. From plan *p* draw a projecting line at right angles to *xy*, and measure off on this line above *xy* the height of *P*,—the point thus found will be the elevation, *p'*. A line from *p'* to the point *o'* where the perpendicular from *o* meets *xy*, will be the elevation of the axis on the new vertical plane.

The elevation may be completed by drawing parallel lines on both sides of the axis at distances equal to the semi-minor axis of the ellipse.

4. *To determine the section of a pyramid when standing on its base and cut by an oblique plane given by its traces.*

Refer to Chapter III. Problems 12 and 14, for constructions which the student should comprehend before working this problem.

Take, e.g., a pyramid with base a regular hexagon. Draw plan and elevation and cutting plane. Assume auxiliary vertical planes containing the slant edges of the pyramid. As these planes contain, and the given plane of section cuts the slant edges, the required points of section in these edges must be in the intersections of the oblique plane with the auxiliary vertical planes. Determine these intersections. The points at which the elevations of the intersections cross the elevations of the slant edges of pyramid will be the elevations of the points of section required. From them determine their plans and join adjacent edges.

The work may be shortened by finding the point in which the axis is cut by the given plane of section. As this point is common to all the intersections of planes to be found, it may be used, when once found, for all planes as a point in the elevation of their intersections. A second point in the required elevation may be readily determined by working from the junction of either the horizontal traces or vertical traces of the planes under consideration according to convenience.

To determine the true shape of the section. 'Construct' the plane of section.

To draw a projection of the solid on its plane of section.

(1) The projection of the section of the pyramid may be found similarly to the section of the prism in Problem 1.

(2) The projection of the vertex on the plane of section as the projection of any one point of the base of the prism on the plane of section in the same problem.

(3) Then lines through this vertex v'' and the points of section just determined, will be the indefinite projections of the slant edges of the pyramid. The points of the base may be fixed by perpendiculars from their elevations to the vertical trace to intersect these indefinite projections of the edges, and when found may be joined consecutively to complete the required projection of the frustum.

The *development* of the solid may be found by describing a circle with radius equal to a slant edge of the pyramid, setting along the circumference the edges of the base as chords, and joining these points to the centre of the circle. The points of section may be marked by dividing the edges in *the same proportion* in which they are divided by the plane of section as indicated either in the plan or elevation.

5. *Take a pyramid standing on its base with any number of unequal sides in its base, and cut it as in the last problem by an oblique plane. Determine the true shape of section, and development of solid.*

6. *Determine the section of a cone by a given oblique plane, and the true shape of the section.*

Divide the circle which is the plan of the base into any number of parts and join each point in the division to the vertex. Consider these lines as the plans of the edges of a pyramid and work accordingly. The section will be an ellipse.

Converse of this Problem.

Given ab *the major axis of the ellipse which forms the oblique section of a right cone by the horizontal plane, given also* vv′ *the vertex; to draw the plan of the frustum.* Fig. 28.

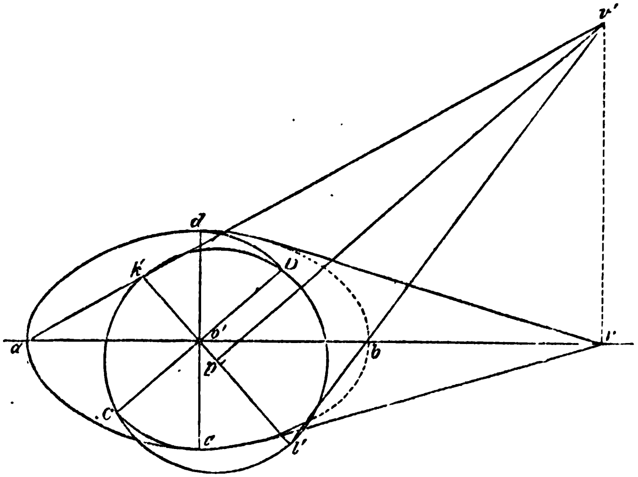
Since the plan *v* in *ab* produced is given, it will be obvious that when the ellipse is obtained, the required plan will be completed by drawing tangents to the ellipse through point *v*. Hence the problem consists chiefly in finding *cd*, the minor axis of the curve. This line is the horizontal trace of a circular section of the cone made through point *o′*, the bisection of *ab*, and forms a chord of that circle.

This circle being perpendicular to the axis *VP* of the cone, is necessarily inclined to the horizontal plane at an angle which is the complement of the angle of inclination of that axis. This circle must therefore be constructed and its chord exhibited.

Take a vertical plane through *V* and *ab*; on it draw *v′a*, *v′b*, the traces of the convex surface; and through *o′* draw *k′l′* making equal angles with these lines; this is the vertical trace and elevation of this circular base.

'Construct' this circle therefore into this vertical plane, as shown in Fig. 28, about *k′l′* its vertical trace and diameter. Draw through *o′* the chord *CD* at right angles to *k′l′*. *CD* gives the length *cd* of the required minor axis.

Fig. 28.



VIII.

TANGENT-PLANES TO CONES.

Problem I.

To determine a tangent-plane to a cone at a given point A *on its convex surface.*

Let V be the vertex of the cone. Join points A and V: the indefinite straight line AV is the generatrix of the conic surface. Determine the 'trace' of the line AV and that of the conic surface, the former will be a point in the latter. At this point determine a tangent to the trace of the conic surface; the plane of this tangent and AV is the tangent-plane required. See Theorem VIII.

Problem II.

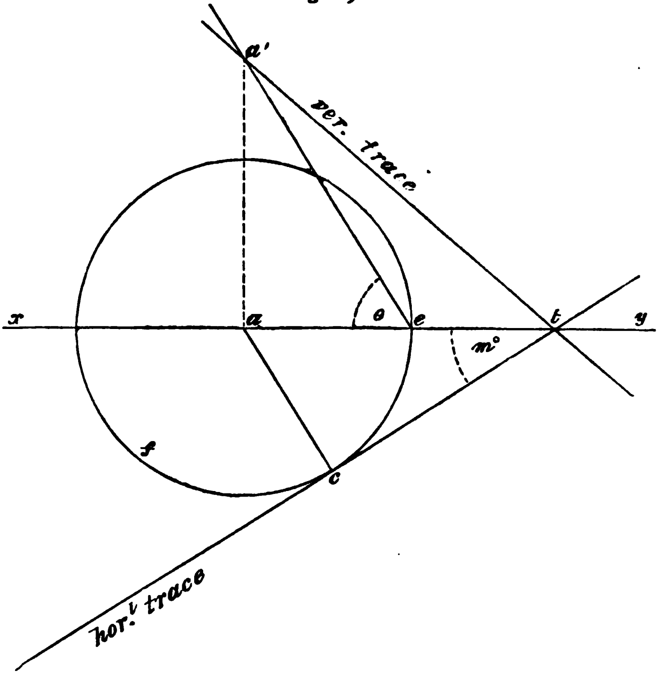
To determine a tangent-plane to a cone, which shall also pass through a point P *which is external to the conic surface.*

If V be the vertex of the cone: join points V and P and obtain the horizontal trace of the indefinite line VP; through this point H draw a tangent to the horizontal trace of the conic surface; the plane of VH and this tangent is the tangent-plane required.

PROBLEM III.

To determine the traces of a plane inclined θ degrees, the horizontal trace of the plane making an angle of m *degrees with* xy. See Fig. 29.

Fig. 29.



Assume a point *aa'* in the vertical plane; make this point the vertex of a right cone generatrix inclined *θ* degrees. Let *cef* be the circular base of the cone. Draw *ct* a tangent to *cef* at *c*, meeting *xy* at *t*, and making the angle *xtc* an angle of *m* degrees; join *a't*, then *cta'* is the required plane; being a tangent-plane to the cone, which it touches in the line *AC*.

PROBLEM IV.

Given plan and elevation of a right conical surface, axis oblique, and vertical angle β degrees. Required a tangent-plane to the surface inclined θ degrees.

Draw elevation of cone on a vertical plane parallel to the axis.

Determine the horizontal trace of cone and draw tangents to it from *v*, the plan of the vertex, to complete plan of cone. See Ch. VII. fig. 28.

Determine the circular horizontal trace of a second cone having the same vertex *vv'* and generatrix inclined θ degrees.

The tangent to the horizontal traces of the two cones is the horizontal trace of the tangent-plane sought.

LIMITS. If *VS* be the generatrix of minimum inclination of the given cone and *VS* be inclined α degrees, then θ lies between α and 90 degrees.

TANGENT-PLANES TO SPHERE.

PROBLEM I.

To determine a tangent-plane to a given sphere centre cc', *at a given point* qq' *on its surface.*

If the sphere rests on the horizontal plane in point *Q*, then that plane is the tangent-plane required.

If *Q* be the highest point on the sphere, the tangent-plane is likewise horizontal.

If Q be situated on the horizontal great circle of the sphere the tangent plane will be a vertical one.

Such plane may make any angle from zero to 90° with the vertical plane of projection; according to the position of point Q on the great circle.

For any other position of the point Q the tangent plane will be an oblique one, making of course an angle of 90° with QC its normal, and therefore inclined at an angle which is the complement of that of the inclination of the line QC. It may therefore be drawn at right angles to QC by the converse of Problem 13, Chap. III.

PROBLEM II.

To determine a tangent-plane to a given sphere centre, cc′, *which shall pass through a given point* qq′ *which lies* without *the spherical surface.*

If tt' or T be the point in which the required plane touches the sphere; then the radius CT is the normal of the plane and is perpendicular to the straight line QT in it; and QTC is a right-angled triangle, having the angle QTC a right angle.

Assume any plane passing through the points Q and C. This plane cuts the sphere in a great circle, a tangent to which through Q determines the point of contact of the tangent plane.

If the triangle QTC revolve about the line QC, it will be observed that the 'locus' of the point T is a small circle of the sphere. The solutions of the problem are therefore as numerous as the points on this circumference, which is the base of a cone with vertex in Q enveloping the sphere.

NOTE. That if the line *QC* be a horizontal one, the *plan* of this circle is a chord of that which forms the plan of the sphere.

Another method.

Determine the cone enveloping the sphere with given point as vertex; see Ch. XI. Prob. 10.

Any tangent to the horizontal trace of the cone will be the horizontal trace of a plane fulfilling the required conditions.

Another method.

Having drawn the circle of contact of enveloping cone, choose any point *T* in this circle and join it to the vertex *V*. This will give *VT* on the surface of the cone. A normal at point *T* is perpendicular to *VT*, and a tangent plane may be determined by drawing a plane to contain *T* at right angles to the normal *CT*.

In this case the ellipse which is the horizontal trace of cone need not be drawn.

PROBLEM III.

To determine a tangent-plane to a given sphere, centre C, *which shall also pass through two given points* P *and* Q, *which are without the spherical surface.*

The problem is a possible one if the straight line which passes through *P* and *Q* does not cut the surface of the sphere.

Determine, as in Problem 2, the 'circle of contact' of a cone enveloping the sphere, and having its vertex in the point *P*.

Determine the circle of contact of a cone enveloping the sphere, and having its vertex in the point *Q*.

Let these circles intersect in points *R* and *S*. Then the planes of points *P*, *Q*, *R* and *P*, *Q*, *S* are tangent-planes to the sphere.

It will shorten the work if when neither *P* nor *Q* lies in the horizontal plane of *C*, a point *V* in the line *PQ* and in that plane be determined and made the vertex of the second cone; since the plan of the circle of contact will then be a straight line (see Note to last Problem) instead of a second ellipse.

A second solution of Problem III.

Take a plane through the centre *C* of the sphere and at right angles to the straight line which passes through points *P* and *Q*. Let *I* be the intersection of this line and plane. Then straight lines through point *I* touching the great circle in which this plane cuts the sphere, give points *R* and *S*: and the planes of the three points *P*, *Q*, *R* and *P*, *Q*, *S* are the tangent-planes.

SOLU

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n of the [illegible]
sphere

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from, taking care that the plan of *B* is so placed that the vertical plane containing the centres of *A* and *B* is parallel to the vertical plane of projection. Draw two elevations of *C*, one tangential to that of *A* and another to that of *B*. Take the horizontal distances between the elevations of the centres of *A* and *C*, and of *B* and *C*, and from the plans of the centres of *A* and *B*, with these distances for radii respectively, describe arcs. The intersection of these arcs will determine the plan of the centre of *C*, the third sphere. Its elevation can then easily be obtained.

(2) To draw the tangent-plane to *A*, *B* and *C*. Determine plan and elevation of axes of cones enveloping *A* and *B*, and *A* and *C*. Lines through the centres of the spheres give these axes. The horizontal traces of these axes will be two points in the horizontal trace of the tangent-plane required, since the tangent-plane to the cones must contain their vertices which are by their position in the horizontal plane. The tangent-plane to the cones is the tangent plane to the spheres they envelop.

PROBLEM VI.

To determine the tangent-planes to three spheres, A, B *and* C, *of unequal sizes and of unequal heights.*

(1) Determine two cones enveloping *A* and *B*, and *A* and *C* respectively. Tangents to the projections of the spheres will determine the projections of the cones. The line passing through the vertices of the two cones will be in the required plane, and its horizontal trace in the horizontal trace of that plane, because the tangent-planes must touch the cones and so contain their vertices.

(2) Determine the two circles of contact of the cones with sphere *A*. The plans of these are ellipses. The points in which the ellipses—see Note to Problem 2, 'tangent-planes to spheres,' by attention to which one of these ellipses becomes a straight line—thus determined intersect will be tangent-points in the required planes, and if these points be joined to points taken in the line passing through the vertices of the cones, the horizontal traces of such lines will be in the horizontal traces of the required tangent-planes. Recollect that three points are enough to determine a plane.

PROBLEM VII.

To determine a cone, with axis vb, v'b' *and vertex* vv' *given, which shall touch a given sphere, centre* cc'.

(1) Determine the horizontal trace of the plane containing given axis of cone and centre of sphere, and make an elevation of this plane on a vertical plane assumed at right angles to it. Observe that this plane cuts the given sphere in a great circle and given cone in a generatrix *VM*, which is a tangent to that circle in *T* or *tt'* the point of contact of the cone and sphere.

(2) Turn this plane into the horizontal plane about its trace, 'constructiug' at the same time points *V* and *B*, and the circle in which the plane cuts the sphere. From *V* draw tangent *VM* to touch the circle turned down, at point *T*, and determine therefrom *t*, the plan of the point in which the cone touches the sphere.

(3) If *V* and *B* be joined, we have the axis 'constructed,' and a line from *B* at right angles to *VB* to meet

VM, the tangent to the circle at *M*, completes the triangle *VBM*, which is the 'constructed' half section of the cone made by the plane containing axis and centre of sphere. The line *BM* is, therefore, the radius of the base of the cone, and may be measured from *b* at right angles to *vb* for the semi-major axis of the ellipse, which is the plan of the base, this axis being horizontal.

(4) To determine the minor axis of the ellipse, make a third elevation of the axis of cone, using the plan of axis of cone for a ground line. In this elevation a section of the cone in true shape will be shown by drawing a line *b's'*, equal to *BM*, from *b'* at right angles to *v'b'*, and completing the triangle *v'b's'*. The side *b's'* is an elevation of the minor axis of the ellipse required. The plan *bs* can then be determined, and the plan of the cone completed.

Obs. The two solids have a common tangent-plane at their point of contact *T*, which touches the cone in the line *TV*. This plane may therefore be drawn at right angles to *CT*.

TANGENT-PLANES TO CYLINDER.

PROBLEM I.

To determine a tangent-plane to a given cylinder at a given point qq′ *on its surface.*

Determine a generatrix of the cylinder at the given point *Q*, by drawing through *q* and *q′* lines parallel to the respective projections of the axis of the solid.

Determine the trace of this generatrix and that of the cylindrical surface. The former is a point in the latter. The straight line through the former, tangential to the latter, is the *trace* of the required tangent-plane.

Obs. (1) If the axis of the given right cylinder be vertical, its horizontal trace is a circle, and its tangent-plane may make an angle varying between zero and 90° with an assumed vertical plane.

(2) If the axis be horizontal, the angle of inclination of the tangent-plane varies between zero and 90°, according to the position of the point *Q*.

(3) If the axis be oblique, the horizontal trace of the surface is an ellipse and the inclination of the tangent-plane varies, according to the position of point *Q*, between 90° and the angle of inclination of the axis.

PROBLEM II.

To determine a tangent-plane to a given cylinder, passing through a given point qq′ *which is* without *the surface of the solid.*

Through the given point Q draw a straight line parallel to the axis of the cylinder, and determine its trace.

Determine the corresponding trace of the cylindrical surface.

A tangent to the latter drawn through the trace of the line is the trace of the required tangent-plane.

PROBLEM III.

To determine a tangent-plane to a given cylinder, which shall have a given inclination θ.

For reasons before indicated, the angle of inclination θ of the tangent-plane cannot be less than the angle of inclination of the axis of the given cylinder.

Determine a plane inclined θ containing the axis of the cylinder, see Chapter III. Problem 9: the plane parallel to this and touching the cylindrical surface is the required tangent-plane.

PROBLEM IV.

To determine a cylinder which shall have a given straight line for its axis and shall touch a given cylinder.

It is obvious that the two cylindrical surfaces will have a common tangent-plane at their point of contact. The problem consists therefore in the determining this point; which will lie in a straight line at right angles to both axes.

Let *AB* be the axis of the given cylinder, and *CD* that of the required one: *AB* not lying in the same plane with *CD*. Determine the line *PQ* at right angles to each of these lines by Problem 23, Chap. III., meeting *AB* in point *P* and *CD* in *Q*.

Let *PQ* pierce the surface of the given cylinder in point *S*, then *S* is the point of contact required, and *QS* is the radius of the base of the required cylinder.

Another method.

Assume any point *P* and draw through it parallels to the two given axes. Determine the traces of these lines and join them for the horizontal trace of the plane containing them.

Another plane parallel to this and touching the horizontal trace of given cylinder will be the tangent-plane of the required cylinder. Its horizontal trace will also touch the horizontal trace of required cylinder.

The generatrices from the two tangent-points in the two ellipses will be the two lines of contact of the cylinders with the tangent-plane, and the point where these lines cross will be the point *T* of contact of the cylinders.

To draw the horizontal trace of required cylinder. Find the horizontal trace of its given axis. Find also the real distance from any point *M* on the axis to the tangent-plane; this will give the radius of the contained sphere. Then the problem is to draw the cylinder with axis and contained sphere given, for which see Chapter XI. Problem 9.

Or, the horizontal trace of required cylinder may also be drawn thus.

Draw from any point in the given axis a perpendicular to the tangent-plane, and find the point where the perpendicular meets that plane. A line through this point parallel to the given axis will be the line of contact of tangent-plane with required cylinder, and *its* horizontal trace will be a point in the horizontal trace of the cylinder. The true length of the perpendicular to the plane will be the semi-minor axis of the elliptical trace. On drawing the minor axis through the horizontal trace of given axis of cylinder at right angles to the plan thereof, we have one axis and a point in the curve, from which data the ellipse can be drawn by means of a piece of paper,

PROBLEM V.

Given two points, B *and* C, *at equal distances from a given plane, as the foci of the elliptic section of a right cylinder which touches the plane: determine the cylinder and its line of contact with the plane.*

The ellipse is perpendicular to the given plane, and touches it in the extremity Q of the minor axis of the curve. This point may be determined by a perpendicular to the straight line BC and the plane, through O the point of bisection of BC; Q being the intersection of this perpendicular and the plane.

Complete the ellipse by making its major axis equal to $BQ + CQ$.

The position of a circular section of the cylinder, perpendicular to the plane with radius equal to *OQ*, can be determined, with one extremity of a diameter coinciding with an extremity of the major axis of the ellipse. See the construction given in Chap. VII. converse of 2.

The straight line, through point *Q*, and the point in which the circle touches the plane, is the required line of contact of the cylinder and its given tangent-plane.

Work the problem:

(1) When the given plane is horizontal,

(2) .... is inclined.

IX.

INTERSECTIONS OF SOLIDS WITH PLANE SURFACES.

NOTE. Developments should be drawn to most of the problems of this Chapter, for which refer to Chapter VII. Problem I.

PROBLEM I.

Two right prisms, with square bases of 2·5 inches side, intersect. One is placed with its edges horizontal and parallel to the vertical plane, and with its lowest face inclined 30° *and its lowest edge* 1 *inch above the horizontal plane. The other stands on its base with one face inclined* 25° *to the vertical plane and with its axis passing through the highest edge of the horizontal prism. Other dimensions at pleasure. Draw plan and elevation.*

Commence by drawing an auxiliary elevation of the horizontal prism, so arranged, that the elevations of both of its bases coincide: that is, draw the square and a ground line inclined 30° to one side and 1 inch below the lowest corner or angle of square. From this determine plan.

The plan of the vertical prism can then be easily placed in position, and will be a square with one side inclined 25° to the ground line of the required elevation.

Then proceed with the required elevation.

For the intersection, the points in which the plans of the edges of the horizontal prism meet the square, which is the plan of the vertical prism, will be the plans of the points of intersection between those edges and the vertical faces; and their elevations can readily be found by perpendiculars from them to the ground line to meet the elevations of the same edges.

The points in which the vertical edges of the one prism intersect the faces of the other will not be evident in plan on account of the vertical position of those edges, but they can be marked off in elevation by transferring their heights, which can be readily taken from the first or auxiliary elevation, to the required elevation.

The proper points must then be joined, and this will best be ascertained by inspection from the auxiliary elevation.

PROBLEM II.

An elevation of the same prisms, similarly arranged, on a vertical plane inclined 40° *to the horizontal edges.*

Use the plan of Problem 1, if convenient for the paper, and draw *xy* 40° with horizontal edges for the elevation.

PROBLEM III.

Let the horizontal prism be intersected by one with base a regular hexagon of 1·5 *inches side standing on its base, and with one edge at least outside the other solid. Other arrangements at pleasure.*

PROBLEM IV.

Plan and elevation of a right prism, with long edges horizontal, bases equilateral triangles, piercing a square pyramid standing on its base. No face of prism to be horizontal or vertical. All horizontal edges to enter pyramid. One side of the base of the pyramd to be inclined 30° *with the vertical plane. Other dimensions and arrangements at pleasure.*

(1) Make an auxiliary elevation at right angles to the horizontal edges of the prism. This elevation of the prism will be the equilateral triangle. Complete plan and second, i.e. the required, elevation of solids preparatory to finding intersection.

(2) The points where the elevations of the edges of the pyramid cross this triangle will be the elevations of the points of intersection of the edges of the pyramid with the faces of the prism. Perpendiculars to the xy from these points will be loci of their plans.

(3) To find the points in which the horizontal edges enter and leave the pyramid, draw in the auxiliary elevation lines from the vertex to the base passing through the elevations of the horizontal edges. These are really lines on the faces of the pyramid passing through the points where the horizontal edges of prism enter and leave the pyramid. Determine these lines in plan. The points where they cross the plans of the horizontal edges of the prism are the plans of the points required.

(4) The proper points must then be joined from inspection.

PROBLEM V.

Take a pyramid and prism of four and five sides in each base respectively, and find intersection. Let the pyramid be placed as in the last problem, and the prism with horizontal edges, and with two entirely free from the pyramid.

PROBLEM VI.

To determine the penetration of a prism standing on its base with a pyramid, when the base of the pyramid is inclined θ°, has no side horizontal, and is not at right angles to the vertical plane.

Draw pyramid and prism, preparatory to find their interpenetration.

The plans of the points at which the edges of the pyramid enter the prism are shown where these edges in plan cross the plan of the prism, and can thence be determined in elevation.

The intersection of the vertical edges of the prism with the faces of the pyramid can be found, *either*, by determining the traces of the planes of those faces and finding their intersection with the vertical edges passing through them, *or*, which is more simple, by drawing lines from the plan of the vertex of the pyramid through the plans of the edges of the prism to the base of the pyramid, and showing these lines, which are really lines on the surfaces of the faces of the pyramid, in elevation. The intersections of the elevations of these lines, with the elevations of the edges of the prism through which they were drawn in plan, give the elevations of the points required.

The proper points must then be joined from inspection, so as to give the whole interpenetration of the two solids.

PROBLEM VII.

To determine the interpenetration of two or more solids with plane surfaces, by means of parallel sections.

Take, for example, two irregular hexagonal pyramids, of assumed but different heights, resting on the horizontal plane and intersecting one another.

Determine their plans only. The faces which intersect can be seen from inspection.

Then, to find the intersection of any two such intersecting faces, draw horizontal lines in each face at the same level.

The points at which these lines *at the same level* cross one another will be points in the actual intersection of the planes of the two faces.

Any two such points being determined, the line of intersection can be drawn through them until it meets an edge bounding one of the faces under consideration.

By continuing this method with all the other intersecting faces, the whole interpenetration will be determined.

X.

INTERSECTIONS OF SOLIDS WITH CURVED SURFACES.

PROBLEM I.

Two cylinders, one horizontal with base 2 *inches in diameter, the other vertical with base* 2·5 *inches in diameter, intersect. The axis of the horizontal cylinder is* 2 *inches above the ground, passes through the axis of the other, and is inclined* 40° *to the vertical plane. Determine plan and elevation.*

Arrange the cylinders in plan and elevation preparatory to finding their intersection.

Determine horizontal lines on the surface of the horizontal cylinder in plan and elevation.

Such lines in plan will be parallel to the plan of the axis, in elevation parallel to its elevation. Assume them according to convenience in plan, and ascertain their heights from an auxiliary elevation made on a vertical plane at right angles to the axis.

The plan of the intersection required will coincide with the plan of the base of the vertical cylinder, the points at which the horizontal lines enter the vertical cylinder being shown in plan where the lines cross the circular base.

The elevations of these points will be determined by perpendiculars from their plans to the ground line, to meet the elevations of the lines in which they must be situated. The elevation of the whole intersection will be completed by joining the elevations of these points.

PROBLEM II.

Find the intersection of two cylinders similarly arranged to those in the last problem, except that the axis of the horizontal one is ·5 *inch from that of the vertical cylinder and its base is* 3 *inches in diameter.*

In this problem the intersection forms one continuous curve.

The work is exactly similar to that in the last problem.

PROBLEM III.

Determine the intersection of a vertical cylinder with base 2·5 *inches, and a cone with base* 2·5 *inches, axis horizontal, passing through that of the cylinder, and inclined* 45° *to the vertical plane.*

From the position of the cone the plan of its base is a straight line, the elevation an ellipse. Determine lines on the conical surface from vertex to base.

For this purpose assume points in the plan of the base, and join them to the vertex. Show these lines in the elevation, by drawing perpendiculars from the points assumed in the plan of the base to the ellipse which is its elevation and thence to the vertex.

Or, make an auxiliary elevation as in Problem 1 for the cylinder and thence obtain heights in the base to be transferred to the final elevation.

The rest of the construction corresponds to that in the preceding problems.

PROBLEM IV.

Arrange an example of cone and cylinder, with the axis of the cone free from that of the cylinder, so that the curve of intersection may be a continuous one.

PROBLEM V.

Arrange an example with cone vertical and resting on its base, and a horizontal cylinder penetrating the cone. Let the axis of the cylinder be free from that of the cone and inclined to the vertical plane, say 45°. Let part of the cylinder be outside the cone.

Determine horizontal lines on the surface of the cylinder in plan and elevation as in preceding problems. Then determine sections of the cone by horizontal planes at the various heights at which these lines on the cylinder are.

In the elevation these sections will be straight lines, being represented by those portions of the lines on the surface of the cylinder which cross the elevation of the cone.

In the plan these sections will be circles described from the plan of the axis of the cone as a centre with the elevations of the corresponding circles for diameters.

The points in plan at which the lines on the cylinder cross the circular sections of the cone on the same level, will give points in the interpenetration of the solids.

These points when determined must be joined from inspection for the plan of the curve, and thence determined in the elevation.

PROBLEM VI.

Plan and elevation of the intersection of a horizontal cylinder with a sphere under the following conditions: sphere with radius 2 *inches and centre* 2·5 *inches high. Cylinder with axis in plan* ·5 *inch from, and on a different level with the centre of the sphere, and inclined* 45° *to the vertical plane. Diameter of cylinder* 2 *inches.*

The construction is similar to that in the preceding problem, the assumed horizontal planes cutting the sphere in circles as in the case of the cone.

PROBLEM VII.

Arrange a cone placed vertically, and a sphere, and find intersection. Let centre of sphere and axis of cone be free from one another.

PROBLEM VIII

Arrange an example of two spheres intersecting, of unequal sizes and unequal heights.

PROBLEM IX.

Intersection of cone and prism.

Prism with base an equilateral triangle, one face inclined 20°, *and with long edges horizontal; cone vertical and resting on its base; one long edge at least of the prism being free from the conical surface, and one at least penetrating it.*

Determine lines on the faces of the prism parallel to its edges. These lines will therefore be horizontal.

Horizontal planes passing through these lines will be shown in elevation by the elevations of the lines, and will cut the cone in circular sections parallel to its base.

Points in the intersection of the solids will be determined in plan where the circular sections of the cone are crossed by the horizontal lines at the same level on the surfaces of the prism.

PROBLEM X.

To determine the intersection of two cylinders, the axes of which are not parallel to either plane of projection.

(1) Draw two lines making any angle, say 120°, with each other for the plans of the axes of the cylinders. Assume an ellipse having its major axis on one of them as the horizontal trace of one cylinder. Complete the plan of this cylinder.

(2) *To determine its elevation.* Use the major axis produced as an *xy*, and draw elevation. This is done by describing a semicircle on the major axis, and setting off from one extremity as a chord the length of the minor. A line at right angles to this chord through the centre of the ellipse is the elevation of the axis of the cylinder. Parallels to this axis from the ends of the major axis of the ellipse complete an elevation of the cylinder which can readily be transferred to any other required plane of elevation.

(3) Arrange a second cylinder similarly on the second line of the angle 120°, so that its convex surface may penetrate that of the former.

(4) *To find their intersection.* Assume a point in plan and elevation, and from it draw two lines parallel to the axes of the cylinders. Determine the horizontal traces of these lines, and join them. The line thus drawn will be the horizontal trace of a plane containing these two lines.

(5) Points in the intersection of the cylinders must then be found by assuming auxiliary planes, cutting the bases of the cylinders and parallel to the plane before found, that is, with their horizontal traces parallel to the horizontal trace of this plane. As each of these cutting planes will contain lines on the surfaces of the cylinders springing from the points where it crosses their bases, and as these lines are all in the same plane, the points in which they actually cut one another will be points in the intersection required.

Any number of points can be found, by assuming a corresponding number of auxiliary cutting planes. The projections of these points must then be joined by hand to get the required curve of intersection.

PROBLEM XI.

To determine the intersection of two cones with oblique axes.

Assume two intersecting cones—by the converse of Problem 5, Chapter XI.—in convenient position.

Planes assumed passing through the vertices of the two cones and cutting their surfaces, will do so in straight lines, the intersections of which lines will give points in the intersection of the two cones.

Draw a line through the vertices of the two cones and find its horizontal trace. The horizontal traces of the assumed cutting planes must pass through this point and cross the two bases. The rest of the work will be similar to that of the last problem.

PROBLEM XII.

Intersection of cone with oblique axis and cylinder with axis inclined to both planes of projection, the axes being free from one another.

Assume cone and cylinder in convenient position.

Draw a line from vertex of cone parallel to the axis of the cylinder. Planes assumed passing through this line and cutting cone and cylinder, will cut the former in lines drawn from points in its horizontal trace to its vertex, and latter in lines on its surface parallel to its axis. The rest of the work will be similar to that in previous problems.

XI.

MISCELLANEOUS PROBLEMS.

1. *Plan and elevation of a cube of* 3 *inches edge, when three of its corners are* 1, 1·7 *and* 3 *inches respectively above the horizontal plane.*

Refer to Chapter III. Problem 21.

Take 1 inch to equal 10 units.

(1) Draw the square *ABCD*, and figure *A* as 1 inch or 10 units, *B* as 17 units, and *C* as 30 units, that is, the respective heights to which these points are to be lifted.

(2) To find a 'horizontal,' in order that the square may be folded about it into the required position.

If *A* and *C*, which are to be at heights of 10 and 30 units, be joined, there will be a point in the line *AC* 17 units high, that is, at the same level as point *B*. Find this point *M* by dividing the line in due proportion, and the line drawn through *B* and *M* will be the horizontal required at level 17 units or 1·7 inches above the horizontal plane.

(3) A vertical plane at right angles to plane of face *ABCD* may now be assumed by taking a line at right angles to *BM* for the intersection, *xy*, of this vertical plane

and the horizontal plane at level 17 units. The vertical trace of plane of face *ABCD* may be easily determined because d' and c' will by the position of the assumed vertical plane be in the vertical trace and their heights are known.

(4) The square *ABCD* is the lowest face of the cube 'constructed' in a horizontal plane at level 17 units, about the horizontal *BM.* The rest of the work therefore is similar to that in former problems.

2. *Plan and elevation of a tetrahedron, edge* 3 *inches, when the three corners of its lowest face are* ·8, 1·5 *and* 2·2 *inches above the horizontal plane.*

3. *Plan and elevation of a cube of* 3 *inches edge, when the indefinite plans of three adjacent edges meeting in point* A *are given.*

Compare Chapter III. Problem 27.

(1) Draw the three given lines as *ab*, *ac*, and *ad* forming 3 obtuse angles at *a.* Because the projections of a perpendicular to a plane are at right angles to its traces, a line drawn at right angles to any one of these edges as to *ab*, will be a horizontal in the plane containing the other two. Draw such a line to cut *ac* in *p* and *ad* in *s.* Then the triangle *pas* is the plan of a right angle which may be 'constructed' about the horizontal *ps* by bisecting *ps* and describing a semicircle on it. The angle in a semicircle being a right angle, point *A* when 'constructed' will be in the intersection of the semicircle with *ab.* If *p* and *A*, and *A* and *s* be joined, the edge of the cube, 3 inches, can be measured from *A* on these lines *Ap* and *As*, or the same produced, and the square can be completed.

(2) The plan and elevation of the whole may be finished by taking a vertical plane at right angles to the horizontal *ps* and working from the square 'constructed.'

4. *Given an oblique plane by its traces, neither being perpendicular to* xy. *Determine the plan and elevation of a tetrahedron resting upon it.*

Determine the inclination of the plane, Chapter III. Problem 5. Take a new ground line at right angles to the horizontal trace, set up the vertical trace in this new vertical plane, complete elevation and plan therein, and thence derive the elevation upon the first or given vertical plane.

5. *An oblique pyramid stands on its base, which is a regular hexagon* ABC...F *of* 1·5 *inches side.* V *is the vertex, the face* VAB *is inclined at* 75° *to the base,* VBC *at* 65°, VCD *at* 60°. *Draw the plan of the solid.* Science Examination, 1870.

(1) Draw the hexagon.

(2) Determine the traces of plane 75°, taking the *xy* at right angles to the horizontal trace.

(3) The same with plane 65°.

(4) In each of these planes determine a horizontal line at the same height. The intersection of these lines gives a point *p* in the plan of the intersection of the two planes.

Another point in the same intersection is *b*, and the line *bp* is the plan of one edge of the pyramid and contains *v*.

(5) Work similarly with planes 65° and 60°, and the

edge *cv* is determined, also containing *v*. The point *v* is fixed by the intersection of the lines *bpv* and *cv*.

6. *Given the plan of a sphere, required to determine the plan of a cube inscribed in it, a the plan of one of its corners, and the direction* ab *or plan of one edge being also given.*

(1) To find the true length of the edge of the inscribed cube. On the diameter of the circle which is the plan of the sphere, mark off one-third of its length from one extremity, and from the point so found erect a perpendicular to meet the circumference. Join the extremities of the diameter to this point in the circumference. The shorter line is the edge of the inscribed cube, whilst the longer would be the edge of an inscribed tetrahedron. *Or*, by computation, if d equals the diameter of the sphere in inches, and e the required edge of the cube, then

$$\sqrt{3} : \sqrt{1} :: d : e.$$

(2) Make an elevation of AB on ab used as a line of level at the height of the centre of the sphere, and determine b. To do this, describe a circle on the part of ab produced which cuts the circle forming the plan of the sphere. Erect a perpendicular to ab from point a to meet the circumference. This gives a'. From a' measure AB as a chord. This gives b', and from b' a perpendicular to ab—which being used as a line of level corresponds for this work to a ground line—determines b.

(3) A line drawn from b through the centre o of the sphere with the distance from b to o measured on it from o will give the plan of g, another point in the face $A \ldots G$.

(4) Determine the plane by its traces containing aa', and perpendicular to ab, $a'b'$. This will be the plane of the

face *A...G* of the cube. To do this, through point a' draw the vertical trace at right angles to $a'b'$, and through *m* where this vertical trace crosses *ab*, the line of level, draw horizontal trace at right angles to *ab*.

(5) 'Construct' this plane and with it points aa', gg'. Join *AG* which will be a diagonal of this face of the cube constructed. Complete the square, and therefrom determine its plan by means of the traces of its plane already found.

(6) The edge *ab* being already drawn, parallels to *ab* and of the same length through the point *a...g* will complete the plan of the solid.

7. *A pyramid,* 5 *inches high, has its axis inclined* 50° *and one diameter AD,* 4 *inches long, of its hexagonal base inclined* 25°. *Draw plan and elevation, and determine the circumscribing sphere.*

(1) If the axis is inclined 50°, the plane of the base will be inclined 40°. Determine this plane by its traces, and draw plan and elevation of the solid resting on it. Refer to Chapter IV. Problem 1.

(2) To determine the sphere. 'Construct' the plane containing the vertex and a diameter of the base of the pyramid. The horizontal traces of the two slant edges of the pyramid springing from this diameter to the vertex will be in the horizontal trace of plane required. Determine the centre of the triangle formed by this diameter and the slant edges of solid thus constructed, and describe circle therefrom passing through the three corners of the triangle, and you have the centre of the required sphere and its radius. If this point be *S*, its projections *s* and *s* can be readily determined.

8. *To determine the centre and radius of a sphere; on the surface of which four given points are situated.*

Care must be taken not to assume these points so that any three are in the same straight line: and the fourth point cannot be situated in the same plane with the other three, unless the four are in the same great circle of the sphere.

If the line joining any two points be bisected by a plane perpendicular to it, since every point in that plane is equidistant from the extremities of the line which it bisects, the plane must contain the centre of the sphere. For the same reason if two other planes be assumed, similarly bisecting any other two of the lines joining the given points, three planes will have been determined, each containing the required centre of the sphere. It will therefore be in their common intersection, that is, in the point in which the three planes intersect.

The length of the radius will be found by means of the right-angled triangle. If the distance from the plan of the centre to that of any point on the surface be taken as the base of a right-angled triangle, and the difference of level between the centre and this point as the perpendicular, the hypothenuse will be the radius required.

9. *Draw the cylinder which would envelop a sphere of* 1·5 *inches radius and* 1 *inch above the ground. Let its axis be inclined* 50°, *and make in plan an angle of* 30° *with the ground line. Show circle of contact and horizontal trace of cylinder.*

(1) Draw sphere and plan of axis of cylinder according to conditions.

(2) Use the plan of the axis as the ground line for a new elevation, in which draw a second elevation of the sphere. A line through the centre of sphere meeting the new ground line at the given inclination will be the elevation of the axis, and parallels to it, tangential to the circle which represents the sphere, will give the major axis of the ellipse in which the cylinder cuts the horizontal plane. The minor axis of the curve will be marked off by parallels to the axis of the cylinder in plan, tangential to the circle which is the plan of the sphere. From the second or auxiliary elevation, the elevation on the original vertical plane can be easily determined.

(3) The circle of contact is shown, in the auxiliary plane of elevation, as a straight line, and from this can readily be determined in plan, and from thence on the original vertical plane.

10. *A cone, with vertex* 5 *inches high, axis inclined* 50° *and in plan making an angle of* 30° *with the ground line, envelops a sphere of* 1 *inch radius and* 1 *inch high. Determine horizontal trace of cone and circle of contact.*

As in the last problem use the plan of the axis as the ground line for an auxiliary elevation. In this auxiliary plane set up the elevation of the axis by drawing a line inclined 50° with *xy* through the point which is the elevation of the centre of the sphere, and determine *vv′* the vertex of the cone 5 inches high. Tangents from *v′* to the circle representing the elevation of the sphere will mark off on *xy* the line which is the elevation of the horizontal trace of the cone. This same line is also the length of the major axis of the ellipse which is the horizontal trace of the cone.

To find the minor axis of the same ellipse, the point must be determined on the circle of contact, through which a line from the vertex to the extremity of the minor axis would pass. In the auxiliary elevation this line will be shown by joining the vertex and the centre of the major axis. The intersection of this line with the straight line representing the elevation of the circle of contact gives the elevation of the point required, through the *plan* of which point a line drawn from the plan of the vertex marks off the extremity of the minor axis sought.

But note that the straight line which in the auxiliary elevation will be the elevation of the circle of contact will not pass, as in the case of the cylinder, through the elevation of the centre of the sphere. It is the line joining the tangent points before determined.

The plan of the circle of contact can be readily determined by previous problems.

The traces on the horizontal plane would represent the shadows of spheres cast by rays of light which are parallel in the case of the cylinder, and which proceed from a fixed point in that of the cone.

The circles of contact would separate the dark and light parts of these spheres.

11. *Plan and elevation of a pentagonal pyramid when one face is vertical and one long edge of that face inclined at* 50°. *Side of base* 1·25 *in., axis* 3·5 *in.* Science Exam. Hon. 1872.

(1) Determine angle θ between base and one face of pyramid.

(2) Determine a line in the vertical plane of projection inclined 50°, and on this line draw a triangular face of the pyramid, one long edge of that face lying in the line.

If the vertical face of the pyramid be assumed parallel to the vertical plane of projection, the triangular face of the pyramid drawn as above will be an elevation of a real face of the solid, and the plan of this real face will be a straight line parallel to xy at any chosen distance in front of the vertical plane.

(3) Assume two points in the short side (i.e. the base), or short side produced, of the triangular face of the pyramid, and make them the vertices of two right cones having their bases in the vertical plane and generatrices inclined to bases at θ degrees.

(4) Draw the tangent plane to these cones. The vertical trace will be a line tangential to the circular bases, and the horizontal trace a line drawn through the horizontal trace of the short side or base of the triangular face of the pyramid and the point where xy is met by the vertical trace.

(5) 'Construct' this plane and the short side of the triangular face with it. On this side 'constructed' describe the pentagon which is the base of the pyramid and determine plan and elevation therefrom.

The projections can be at once completed by joining the proper points, all of which are thus determined.

12. *A right pyramid, having a hexagon of* 1·25 *in. side for its base, and an axis of* 3·5 *in., lies with one edge on the horizontal plane, and a face containing that edge is inclined at* 25°. *Draw plan and elevation.* Science Exam. Hon. 1870.

(1) Determine the angle a^0 between the base and one face of the pyramid.

(2) Determine plane of 25° by its traces.

(3) 'Construct' this plane and draw in it a triangular face of the pyramid with one of its equal sides lying in the horizontal trace, and thence determine plan.

(4) Produce the base of the triangular face and construct it.

Note. The line thus constructed will be the produced base of the isosceles triangle first drawn.

(5) Describe a circle of any convenient radius touching this line and mark P, the point of contact. Assume this circle to be the base of a cone with generatrix inclined α^0, and find the vertex, V.

(6) Find the projections pp', vv' of PV, and the horizontal trace of a straight line drawn from V to P.

The line drawn through this trace and the point where tangent line to base of cone at point P meets the horizontal trace of plane 25°, will be the horizontal trace of a plane inclined α^0 to this plane, and will contain the base of the triangular face of pyramid determined in (3), and hence is the plane of the base of that pyramid. This plane can now be constructed and the projections determined as in previous problems.

It may be remarked that the line PV is that in which the plane of the base of the pyramid touches the cone.

13. *The two extremities* A, B, *of one edge of a cube are at heights of* ·4 *in.*, 1·6 *in. above the paper. The centre* O *of the solid is at a height of* 1·4 *in. Draw the plan of the solid. Edge* = 3 *inches.* Science Exam. Hon. 1870.

The section of the cube made by a plane passing through the edge AB and centre O of the solid is a rectangle, two sides of which are edges of the cube and two diagonals of the faces.

By the conditions, the heights of three points—two extremities of one short side, and the centre—of this rectangle are given.

(1) Determine plan and elevation of this rectangle as in Problem 1.

(2) Determine a plane at right angles to one of the short edges of the rectangle and containing one of the long edges of the latter. This condition is complied with if the plane be drawn through one extremity of a short side and perpendicular to it. Converse of Chapter III. Prob. 13.

Or the horizontal trace of the plane may be drawn at once through the horizontal trace of one long side of the rectangle at right angles to the short sides of the latter.

(3) Construct the plane thus drawn, which is the plane of one face of the cube, and with it the long side of the rectangle.

The latter being a diagonal of a face of the cube, the plan of the solid can be completed therefrom as in previous problems.

14. *An irregular pyramid has for its base a triangle* ABC*:* AB = 3 *in.;* AC = 3·5 *in.;* BC = 4 *in.: the plan* d *of the fourth corner projected on the plane of the base is* 2 *in. from* A*;* 1·5 *in. from* B*; the true length of the remaining edge* CD *is* 3·7 *in. Draw the plan of this pyramid, when standing on its base, and an elevation on a plane parallel to the edge* AD. *Determine the lengths of the edges* AD, BD, *and the height of* D *above the base.*

a. Determine the inclinations of the faces ABD, ACD, *to the plane of the base.*

β. Draw a plan of this pyramid when the edge CD *is vertical.* Science Exam. 1868.

(1) Draw the triangle *ABC*. The point *d* will be found by describing a circle of 2 in. radius from *A* as a centre and another of 1·5 in. radius from *B*; the point where the circles intersect on that side of *AB* nearest to *C*, is the point required.

(2) Join *dC*, and from *d* draw a perpendicular to this line. A circle of 3·7 in. radius described from *C* as a centre will cut this perpendicular at *D*, and *Dd* will be the height of the vertex of the pyramid.

(3) To determine the lengths of the edges *AD*, *BD*, draw from *d* lines perpendicular respectively to *Ad* and *Bd*. Make these perpendiculars equal to height of vertex = *dD*, and complete the right-angled triangles *AdD*, *BdD*; the hypothenuses *AD*, *BD* of these triangles are the edges required.

(4) For the required elevation, *xy* must be taken parallel to the plan *dA* of the edge *AD*. The rest of the work presents no difficulty.

Note. The elevation of *AD* on this vertical plane is in true length.

(5) For the inclination of the faces *ABD*, *ACD*, elevations may be made on vertical planes at right angles to the lines *AB* and *AC* respectively.

(6) For the plan (β) make an elevation on a plane parallel to *CD*, and from this elevation deduce the plan required, by taking *xy* at right angles to the elevation *c′d′* of *CD*.

15. *Plan and elevation of a tetrahedron inscribed in a sphere of 2 inches radius. Two adjacent edges of the solid inclined at angles of 20° and 30° respectively. Centre of sphere 3 inches above horizontal plane.*

(1) Find edge of tetrahedron inscribed in a sphere of 2 inches radius. See Problem 6, page 124.

Note. The edge of tetrahedron is a diagonal of face of inscribed cube.

(2) Draw an equilateral triangle with sides equal to edge of inscribed tetrahedron, and determine the diameter of a circle circumscribing this triangle. Set off the diameter of this circle as a chord of a great circle of given sphere, and find the length of a perpendicular from the centre of great circle to the chord drawn. Call Ox the length of this perpendicular; then Ox is the length of a perpendicular from centre of given sphere to plane of face of inscribed tetrahedron.

(3) Determine on a convenient part of the paper the plane of a face of any tetrahedron having two adjacent edges of that face inclined as above. Chapter III. Problem 20.

(4) Draw from centre, O, of the given sphere, a line perpendicular to the plane determined in (3) and cut off on this line from the centre of given sphere a length equal to Ox. A plane through the point x at right angles to the perpendicular Ox, will be parallel to the plane of the face of the auxiliary tetrahedron first drawn, and will also be the plane of one face of the solid required.

(5) 'Construct' this plane with the circle in which it cuts the given sphere.

(6) Inscribe in the circle constructed the triangular face of the required tetrahedron. To do this:—draw three tangents to the circle parallel to the three sides of the triangular face of the auxiliary tetrahedron first drawn for the purpose of determining the plane of a face of that

solid. The three tangents constitute an equilateral triangle circumscribing the circle. Draw lines from the three angles of this figure to the centre of the circle, and join the points where these lines cut the circumference. This will be another equilateral triangle with sides parallel to the last inscribed in the constructed circle, and will be, moreover, the constructed plan of the face of the required inscribed tetrahedron, from which the projections can be at once completed.

16. *The centre* O *of a sphere of* 2 *inches radius is* 3 *inches above the plane of the paper. A point* a *on the surface,* 4·25 *inches high, is the plan of one corner,* A, *of an octahedron inscribed in the sphere. The edge* AB *is inclined* 45°. *Draw the plan and an elevation on a vertical plane inclined* 30° *to the plan* ao.

(1) Determine the edge of the octahedron. This will be equal to the side of a square inscribed in a great circle of the given sphere.

If r = radius of sphere, x = side of inscribed octahedron, then

$$r : x :: 1 : \sqrt{2}.$$

(2) Make plan and elevation of sphere, and determine the plan and elevation of A on its surface. To do this:—draw, on the elevation, a line at the height of 4·25 inches parallel to xy and cutting the elevation of the sphere. This may be considered as the vertical trace of a horizontal plane passing through sphere at the assumed level. The section of the sphere by this plane will be a circle shown in true form in plan, and as a straight line in the trace of the plane in elevation. The point A is on the circumference of the circle constituting the plane section of sphere, and its pro-

jections *a*, *a'* are on the corresponding projections of the section. To find *a*, draw a line from *o*, the plan of the centre of the sphere, making an angle of 30° with *xy*, and produce it to meet the circle which is the plan of the section. The point of intersection is *a*. *a'* is at once found by projecting *a* to the straight line drawn at 4·25 inches above *xy*.

(3) Make *A* the vertex of a cone, axis vertical, generatrix inclined 45° and equal in length to the side of the octahedron (1).

The base of this cone in elevation is a straight line and in plan a circle, with radius equal to that of the given sphere, described from the point *a* as centre. The section of the sphere by the plane of the base of the cone produced, is another circle. Show the latter in plan. Either of the two points where the plan of the circular base of the cone meets the plan of the circle cut from the sphere by the plane of the base of the cone produced, may be taken as the plan *b* of the other extremity *B* of the edge *AB*.

(4) Determine *b'* and draw *ab*, *a'b'*. These are the projections of the edge *AB*.

(5) Determine the plane containing the straight line *AB* and the centre of the sphere *O*. Construct it and complete the square *ABCD*, which will be inscribed in a great circle of the sphere.

(6) Determine the plan *abcd* and a perpendicular to the plane of *ABCD* from *O* meeting the surface of the sphere on both sides of the plane at *P* and *R*.

(7) Complete plan of solid by joining *p* with *abcd* and *r* with the same points, and elevation by joining *p'* and *r'* with *a'b'c'd'* as before.

17. *A parallelogram, sides 3 inches and 2 inches, included angle 60°, is the plan of a square. Determine the side of the square and the inclination of its plane.* Science Exam. Hon. 1871.

(1) Draw parallelogram and inscribe in it an ellipse.

This ellipse will be the plan of a circle inscribed in the square of which the parallelogram is the given plan, and which circle will necessarily lie in the same plane with the square; that is, in the plane whose inclination we have to find.

(2) Determine the major and minor axes of this ellipse.

The former is a horizontal line lying in the plane of the square, and is therefore the diameter of the circle of which the ellipse is a plan. This latter property carries with it two others, viz. that of being the diameter of the circle inscribed in the square whose side is required, and also that of being *equal* to that side.

(3) To determine the inclination of the plane, it is only necessary to make a projection of the minor axis on a vertical plane assumed at right angles to the major axis. The plane of the square and the assumed vertical plane are perpendicular, hence, the elevation of the minor axis of the ellipse will fall in the vertical trace of the plane of the square and its angle of inclination with xy will therefore be the same as that of the plane.

The student will require no further assistance in determining this plane or in constructing it with its contained square for the purpose of verifying the side found from the major axis of the ellipse.

18. *A tetrahedron, of 3 inches edge, stands with one corner on the horizontal plane, so that the plans of the two edges which*

meet at that corner are 1·9 *in. and* 1·5 *in. respectively. Draw the plan of the solid.* Science Exam. Hon. 1871.

(1) Determine the plane of the face containing the edges whose lengths in plan are given.

To do this:—draw a line and make it equal in length to either of the plans of the given edges. Show an elevation of this edge on a parallel vertical plane. For this purpose the *xy* must be taken parallel to the line drawn, and should be assumed at a convenient distance therefrom. An elevation on this plane will show the edge in true length, i.e. 3 inches.

This determines one edge of the solid. The other is of the same real length and inclined at a constant angle of 60° to the first. It is plain, therefore, that the second will lie somewhere on a conical surface having the first for an axis and the second for a generatrix; that is, on a cone whose vertex is at the point where the first edge meets the horizontal plane,—vertical angle 120°, generatrix 3 inches, and axis coincident with the edge whose projections we have found.

Determine plan and elevation of this cone. The elevation of the base will be a straight line bisecting at right angles the vertical projection of the edge first found, and the plan, an ellipse, having its centre at the middle point of the plan of that edge.

Describe a circle from the vertex of the cone in plan with a radius equal to the length of the plan of the second edge; and from either point common to circle and ellipse draw lines to the two extremities of the first edge.

The triangle thus drawn is the plan of the face of the tetrahedron containing the two given edges. From this the plane of the face can be at once drawn.

(2) Construct this plane and complete plan of tetrahedron as in previous examples.

19. *An irregular triangular pyramid has its edges* AB = 2; AC = 2·5; BC = 3; AD = 3·33; BD = 3·45; CD = 3·84 *inches. Determine the plan of the sphere circumscribing this solid when the plane of the face* ABC *is inclined* 40° *and one edge* AB *of that face* 30° *to the horizontal plane.*

(1) Determine the plane of 40° and the line inclined 30° lying in it.

(2) 'Construct' the plane with the line of 30°, and on the line so constructed describe the triangle *ABC*.

(3) On the side *AB* describe another triangle *ABD*. Imagine the triangle *ABD*, which is a face of the pyramid, to revolve about *AB* as an axis; the point *D* will describe a circle in a plane perpendicular to *AB* and the locus of *D* about the axis *AB* will be, in plan, a straight line drawn from the vertex *D* of the triangle *ABD* at right angles to *AB*.

Draw the plan of this locus and produce it indefinitely.

(4) On the side *BC* describe a triangle *BCD*.

A line from *D* perpendicular to *CB* will be the plan of the locus of *D* about *CB* as axis.

Draw this line and produce it to cut the line determined in (3).

(5) The point *d* where the loci intersect, in plan, is the plan of the vertex *D*. The height of this point can be readily determined by an elevation on a vertical plane parallel to one of the slant edges.

The plan and elevation of the solid on the plane of 40° present no difficulties. See Chapter IV.

The four points *A*, *B*, *C*, *D*, situated at the solid angles of the pyramid, are those through which the spherical surface has to pass, and the centre of this sphere is found as in Problem 8 of this Chapter.

20. *Draw a triangle* ABC *having* AB = 2·7, BC = 3·2, AC = 2·3 *inches. The points* A, B, C *are the stations of three observers who at the same moment take the altitude of a point* P *in space. The angle by* A *is* 33°, *by* B 37°, *and by* C 46°. *Show the plan of the point and find its height in inches above the plane of* ABC. Science Exam. Hon. 1872.

The point *P* is the common intersection of three inverted cones with axes vertical, vertices at the points *A*, *B*, and *C*, and generatrices inclined to the planes of the bases at angles of 33°, 37° and 46° respectively.

To work the problem :—

(1) Make elevations of the cones on a vertical plane and across the elevations draw a series of lines at convenient distances apart each parallel to *xy*. These lines are to be regarded as the vertical traces of a series of horizontal planes at the levels shown by the lines in elevation.

(2) The sections of the cones by the horizontal planes are circles, which are shown in true form in plan. Determine the plans of the sections of any two of the cones. The points where the circles which are the plans of the sections of the two cones under consideration by the same horizontal plane cut one another are points in the plan of the intersection of those cones, and a curve drawn through these points will be the plan of that intersection.

Similarly the plan of the intersection of one of the above cones with the third can be found.

The point where the two intersections in plan cross is the plan p of the point P.

(3) The elevation p', and therefore the height of P, can be easily determined thus: From the plan of the vertex of one of the cones, as A, describe a circle to pass through p. This circle is the plan of a section of the cone whose vertex is A by a horizontal plane passing through P. A line at right angles to xy and a tangent to the circle will cut the slant side of the cone in elevation at a point the height of P above xy.

21. *The plane of a circle 2·5 in. diameter is inclined at 25°. Its centre is 2·8 in. above the horizontal plane. Determine the sphere which resting on the horizontal plane has this circle on its surface.* Science Exam. Hon. 1873.

(1) Determine plane of 25° and the projections of the circle lying therein.

A straight line drawn from the centre of the given circle at right angles to its plane will pass through the centre of the required sphere.

(2) Draw projections of a line at right angles to given plane from centre of given circle and find the horizontal trace of this line.

(3) Draw a vertical plane containing the line determined in (2). This plane will be perpendicular to plane 25° and its horizontal trace will pass through the horizontal trace of the perpendicular to this plane (2) and will also be at right angles to the horizontal trace of the given plane.

The vertical plane will cut the given circle at two points which are the extremities of a diameter, shown in plan as the minor axis of the ellipse which is the plan of the circle. These points are, moreover, two points in a great circle of the sphere which has the given circle on its circumference and which rests on the horizontal plane at a point in that great circle.

The plane of this great circle is vertical and coincides therefore with the vertical plane just drawn. The horizontal trace of this plane is a tangent at the point of the great circle mentioned above as that in which the sphere touches the horizontal plane.

(4) 'Construct' the vertical plane and with it the two points.

(5) Determine a circle passing through these points and touching the horizontal trace of the vertical plane (3). The centre of this circle is that of the sphere constructed, and its radius that of the sphere required.

Find plan and elevation of the centre and describe circles therefrom with the radius found.

The circles so drawn are the required projections of the sphere.

22. *The axes of two equal cylindrical surfaces are inclined in opposite directions at* 25°, 50°; *the line perpendicular to both these axes is* 3 *inches long and inclined* 35°; *the two surfaces touch each other in one point only. Draw plan of cylinders showing point of contact. Elevation at pleasure.*

Science Exam. Hon. 1869.

(1) Determine projections of the axes and their common perpendicular by Problem 22, page 73.

(2) Bisect the plan and elevation of the line of 3 inches which is perpendicular to the axes. The points of bisection are the projections of the point in which the cylinders touch.

(3) The plans of the cylinders can be completed by making auxiliary elevations of the axes on vertical planes assumed containing these axes. Lines parallel to the elevations of the axes and in the same vertical planes with them, at distances of 1·5 in. on each side, meet the horizontal plane in points on the plans of the axes produced which are the extremities of the major axes of the elliptic sections made by the horizontal plane. The semi-minor axes of the same sections are equal to 1·5 in., and hence the ellipses can be drawn and the plans and elevations completed.

23. *Three spheres* A, B, C *of* 2·3, 1·9 *and* 1·3 *inches diameter rest upon the horizontal plane, each touching the other two. Determine plan and elevation of a fourth sphere,* ·75 *inch radius, touching all three.*

(1) Draw plan and elevation of the three spheres *A*, *B*, *C* as in Problem 5, page 100.

(2) Determine an elevation of two of the spheres, as *A*, *B*, on a vertical plane parallel to the line joining their centres. The elevation will be two circles touching at a point on their circumferences which is a projection of the point in which the spheres themselves touch.

(3) Draw a third circle of ·75 inch radius touching each of the two circles determined above. Consider the circle last drawn as a projection of the sphere of ·75 inch radius which is required to touch all three given spheres. From the drawing it will be apparent that the elevation drawn is that of a sphere ·75 inch radius touching two only, viz. *A* and *B*, of these spheres. Conceive the sphere of ·75 inch radius to roll round the two spheres *A*, *B* and preserve its contact with each during the revolution. The centre of the revolving sphere will describe a circle in a plane at right

angles to the line *AB*, which circle in elevation will be a straight line drawn from the centre of the circle of ·75 inch radius perpendicular to *a′b′* and produced to an equal distance on the opposite side of this line.

Note. The circle described by the centre of the revolving sphere is the locus of the centres of all spheres of ·75 inch radii which touch each of the spheres *A*, *B*.

(4) Determine plan of this circle. The plan will be an ellipse with its major axis at right angles to *ab*.

(5) Proceed similarly with one of the two spheres taken above and the third *C*, and determine the projections of the locus of the centre of a sphere of ·75 inch radius in simultaneous rolling contact with both. The plan of this locus will likewise be an ellipse, and the two points where the loci intersect will be shown in plan by the intersection of the ellipses.

The points thus determined are the centres of spheres of ·75 inch radii which satisfy the required conditions.

For practice another example may be taken in which the three spheres *A*, *B*, *C* are at different heights above the horizontal plane and not in contact, taking care not to remove the centres so far apart as to render the problem impossible.

24. *Three equal spheres, diameters 2 inches each, have their centres at the angl s of a vertical equilateral triangle of 4 inches side, with one side horizontal. Draw a tangent plane to the three spheres—*

(a) *To pass above the two lower and under the upper sphere.*

(b) *To pass above one of the lower and under the other two spheres.*

For *a.* Draw auxiliary elevation on a vertical plane at right angles to the plane of the triangle. A tangent to the two circles which are the elevations of the three spheres can be drawn for the vertical trace of the required plane. The horizontal trace is perpendicular to xy.

For *b.* Draw elevation as a vertical plane parallel to the plane of the triangle. Determine a cylinder containing the upper and one of the lower spheres, and a second cylinder parallel to the first cylinder and containing the third sphere.

The horizontal trace of required plane can now be drawn in proper position as a tangent to the two ellipses which are the horizontal traces of the cylinders.

The tangent lines on the cylinders can be shown by drawing generatrices from the tangent points of the ellipses.

The tangent points on the spheres by drawing perpendiculars to the tangent plane from their centres. These points are fixed where the projections of the perpendiculars to the plane are cut by the tangent generatrices.

25. *Plan and elevation of a right pentagonal pyramid under the following conditions:—Plane of face* VAB *inclined* 50° *to horizontal, and* 75° *to vertical planes, one long edge* VA *of face inclined* 25° *and lowest corner,* A, 1 *inch above the horizontal plane. Other dimensions at pleasure.*

(1) Determine angle α between the base and one of the faces.

(2) Determine the face *VAB* lying in a plane inclined 50° to horizontal and 75° to vertical planes. Chapter III. Probs. 7 and 19.

(3) Determine a plane containing the edge *AB* of the face *VAB* and inclined to the plane of that face at the angle α. Chapter III. Prob. 15, *converse*.

(4) 'Construct' the plane determined in (3), and on the edge *AB* contained therein describe the pentagonal base of the given pyramid. Determine the projections of the base and join *abcde*, plans of the corners of the base, with v, and $a'b'c'd'e'$ with v', for the complete projections of the solid.

26. *A triangle* ABC *is the base of a pyramid whose vertex is* V. AB = 5 *inches;* BC = 4 *inches;* CA = 3 *inches. The angle* AVB = 60°; BVC = 45°; CVA = 40°.

Find the plan of the point V *and its height above the base.*

This problem is a simple case of the intersection of surfaces of revolution.

(1) Draw the triangle *ABC*, and on *AB* describe a segment of a circle containing an angle of 60°, on *BC* one containing an angle of 45°, and on *CA* a third containing an angle of 40°. Euclid, Book III. Prop. 33.

If the segments be supposed to rotate about the lines *AB*, *BC*, *CA* as axes, surfaces of revolution will be generated the common intersection of which will determine *V*.

(2) Find the intersection of any two of these surfaces. To do this:—let *AB* and *BC* be the two axes of the surfaces whose intersection we require. From *B* describe a circle with any radius cutting the segment described on *AB* in p, and the segment on *BC* in q.

If the segment on AB revolve about that line as an axis, the point p will describe a circle in a plane perpendicular to AB. The *plan* of the locus of p is, therefore, a straight line drawn from p at right angles to AB and produced to an equal distance on the other side of that axis. The locus of p itself is a circle, and may be regarded as the base of a cone with generatrix equal to Bp, and axis in the line AB.

Similarly the plan of the locus of q will lie in the perpendicular from q to BC, and the real locus may be considered the base of a cone with generatrix $= Bq = Bp$.

The point of intersection of the plans of the loci is a point in the plan of the intersection of the two surfaces. That this is the case the student can satisfy himself by noting that the point is the plan of the intersection of the circular bases of cones whose generatrices are equal. Or, the point B may be considered as the centre of a sphere radius $= Bq = Bp$, and the loci of p and q plane sections of this sphere.

In like manner any number of points may be found in the plan of the intersection and the latter line drawn through them.

(3) Find the intersection of the third surface with one of the foregoing. The point where the plans of the two intersections cross is the plan of the vertex V.

It may be remarked that there are two points answering to V—one above, and one below the plane of the triangle ABC.—The latter is neglected.

(4) Complete plan by drawing lines from *v* to *A*, *B* and *C*, and find the height of *V*. To do this:—draw a line from *v* at right angles to (say) *AB* and produce it to meet the segment on this line at the point *x*.

Then *Bx* will be the real length of the edge of which *vB* is the plan, from which data the height of *V* can be at once determined.

XII.

SPHERICAL TRIANGLES.

CASE I.

Given the three sides a, b *and* c*; to determine the three angles* A, B *and* C.

Note. In 'spherical triangles' the angles A, B, and C are those which are opposite to the sides a, b, and c, each to each; namely A to a, B to b, and C to c.

Let the three sides be developed on the horizontal plane of one of them, viz., b.

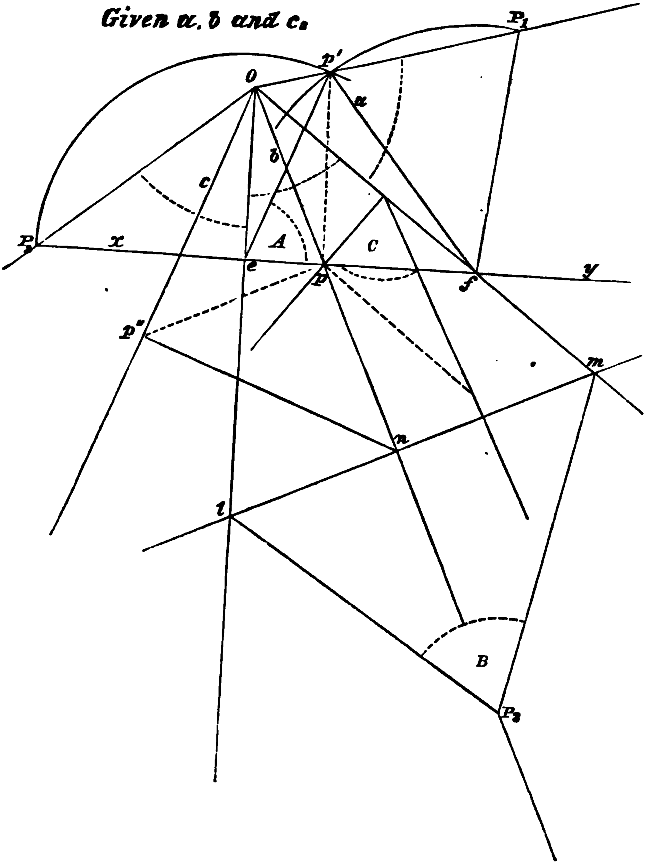
Take a plane of elevation xy at right angles to Ol, the common edge of the sides c and b; and let it meet the second edge of side c in point P_2, and cut Ol in e and Om in f.

Make OP_1 on the second edge of a equal to OP_2; since OP_1 and OP_2 are really the same line, viz., the common edge of sides c and a.

Then eP_2, ef, fP_1 are the sections of the three sides by the plane xy.

Construct this section $ep'f$, in this plane; taking the centre e and radius eP_2, and centre f and radius fP_1, and describing arcs intersecting in p': join $p'e$ and $p'f$. Then p, the plan of P, is determined in xy from p', and Op is the plan of the edge OP on the plane of b. Angle $p'ef$ is the angle A. The plane through p at right angles to Om gives the angle C. And the third angle B is determined by a plane through P, in space, at right angles to the third edge OP, in space, by Problem 15, Chap. III. lm is the horizontal trace of this plane, which is that of the profile angle of B.

CASE I.



Given a, b and c.

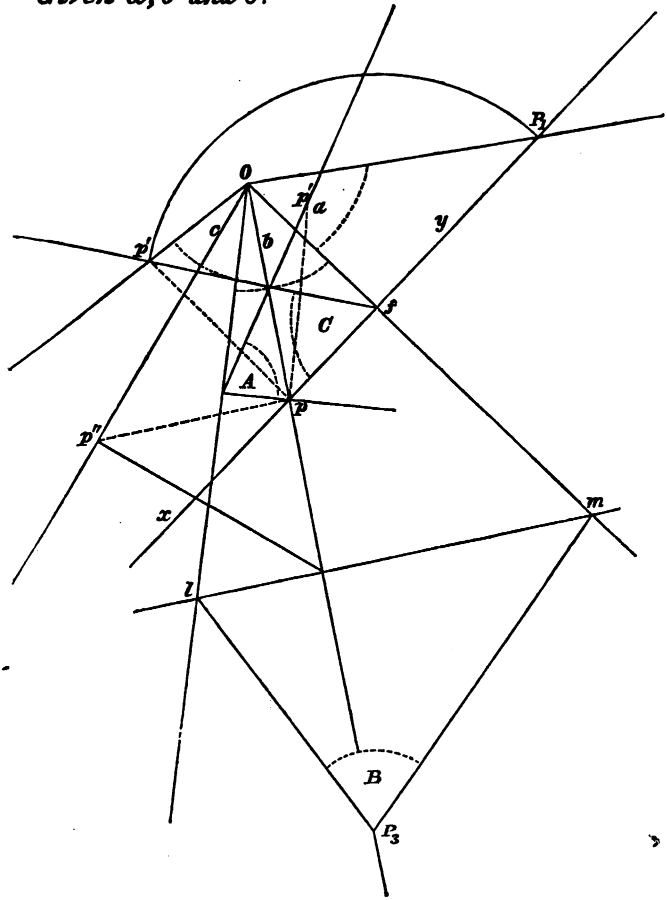
CASE II.

Given the sides a, b, *and the angle* C*: to determine* A, B *and* c.

Let the side *a* be developed on the horizontal plane of *b*: and since the angle *C* between these sides is known, a plane *xy* at right angles to their intersection *Om* exhibits this angle *pfp'*; *fp'* being the vertical trace of the side *a*, the length *fp'* being made equal to fP_1, *p* in *xy* determined from *p'* gives *Op*, which produced is the indefinite plan of the third edge *OP* on the plane of *b*. A plane through *p* at right angles to the edge *Ol* gives the angle *A*, and enables us to 'construct' the third side *c* about *Ol* its horizontal trace. A plane through *P* in space, and at right angles to the edge *OP* in space, and having *lm* for its horizontal trace, will determine the profile angle of *B*.

CASE II.

Given a, b and C.



CASE III.

Given the angles B *and* C *and side* a, *that is, two angles, and their adjacent side: to determine the angle* A *and the sides* b *and* c.

On the horizontal plane take the side *a*, of which *Ol*, *Om* are the edges.

Take the plane *xy* at right angles to *Ol*.

The vertical trace of the side *c* on this plane forms the angle *B* with *xy*, which angle being known this trace can be drawn.

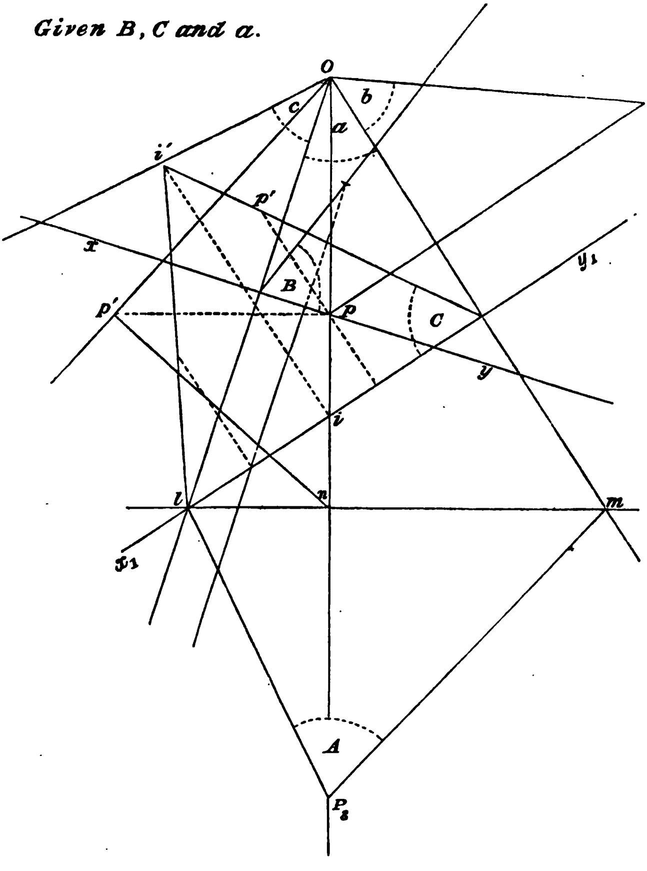
On a plane x_1y_1, at right angles to *Om*, the vertical trace of side *b* forming the known angle *C* with x_1y_1 can be drawn.

By the aid of the elevation on *xy* determine the vertical trace of side *c* on plane x_1y_1; the two vertical traces on the latter plane intersect in a point *i'*, whence *i* is found in x_1y_1 and *ii'* is a point in the third edge *OI*, of which draw the indefinite *plan OI*.

The faces *b* and *c* can now be readily 'constructed' on the plane of *a* by rotation on *Om* and *Ol* respectively. Angle *A* is found by Problem 15, Chapter III. as before, by a plane through point *P* or *I* and perpendicular to *OI*.

CASE III.

Given B, C and a.



CASE IV.

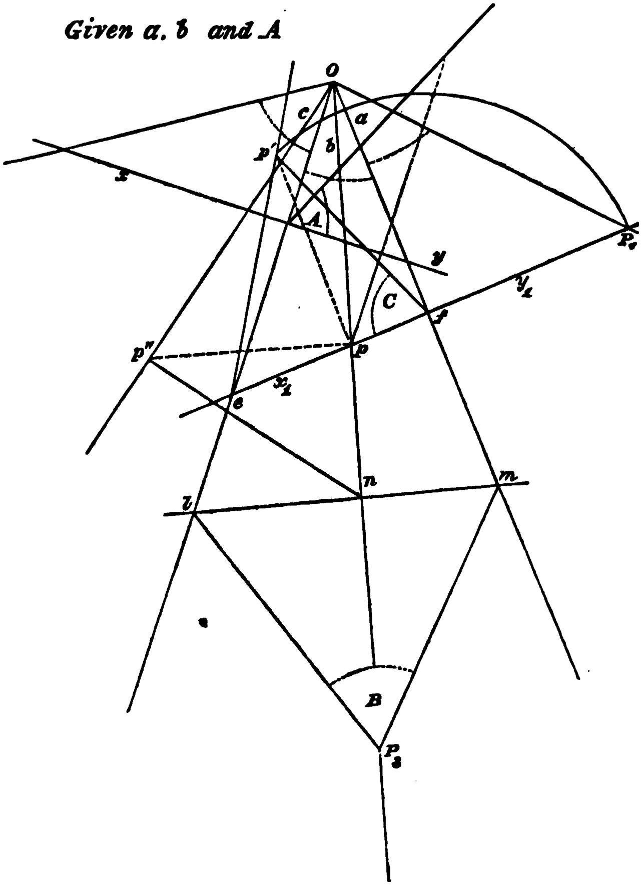
Given two sides a *and* b, *and the angle* A (*that is, two sides, and an angle opposite to one of them*): *to determine the angles* B *and* C, *and side* c.

Assume the given side b on the horizontal plane, edges Ol and Om and side a 'constructed' into that plane about Om.

On plane xy at right angles to Ol make the given angle A with xy, the second line of this angle is the vertical trace of side c on this plane.

On a second plane x_1y_1 assumed at right angles to Om determine the vertical trace ep' of the side c by the aid of the former elevation: cut this trace in p' by the circular arc described on plane x_1y_1 by point P_1 on the third edge: join $p'f$, it is the vertical trace of side a; obtain p in x_1y_1, then pp' is a second point determined in the third edge OP of the spherical triangle. Draw the indefinite plan Op and obtain the third angle B as before by Problem 15, Chapter III.

CASE IV.



Given a, b and A

CASE V.

Given angles B *and* C *and a side* b *opposite to one of them: to determine sides* a *and* c, *and the third angle* A.

Let the side *b* be drawn on the horizontal plane, and revolved about its edge *Om* until it makes the angle *C* with that plane.

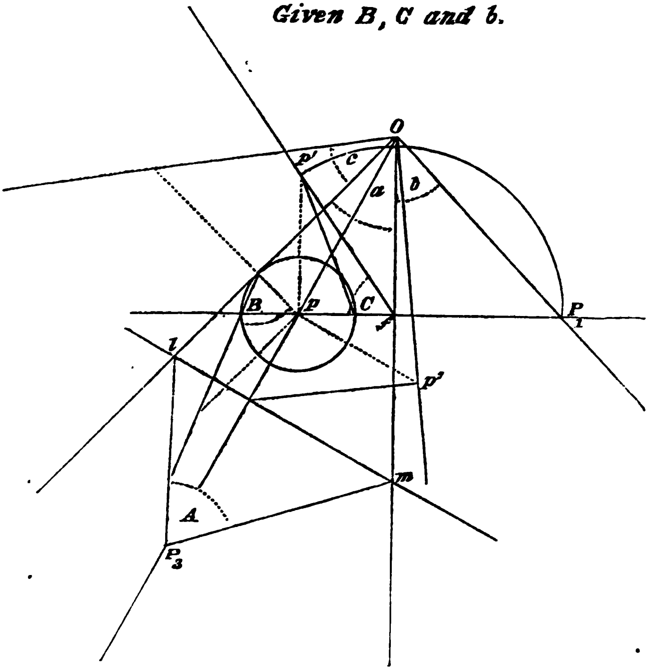
The indefinite line *fp'* will then be the vertical trace of side *b* on the plane *xy* of the profile angle of *C*; which is, of course, perpendicular to *Om*: and *fp'* on this trace, taken equal to fP_1, gives the elevation *p'* of a point *P* in the second edge of side *b*. *p* in *xy*, obtained from *p'*, determines the point *pp'* in this second edge: and *Op* is the *plan* of that edge.

Determine a plane to contain the line *OP*, in space, just found, and inclined at the known angle of *B*, by Problem 9, Chapter III.: *Ol*, the horizontal trace of this plane, is the third edge of the required spherical triangle.

The side *c* can now be 'constructed' on the plane of *a* about *Ol*. The angle *A* may be determined as before, by a plane at right angles to *OP* through point *P*.

CASE V.

Given B, C and b.



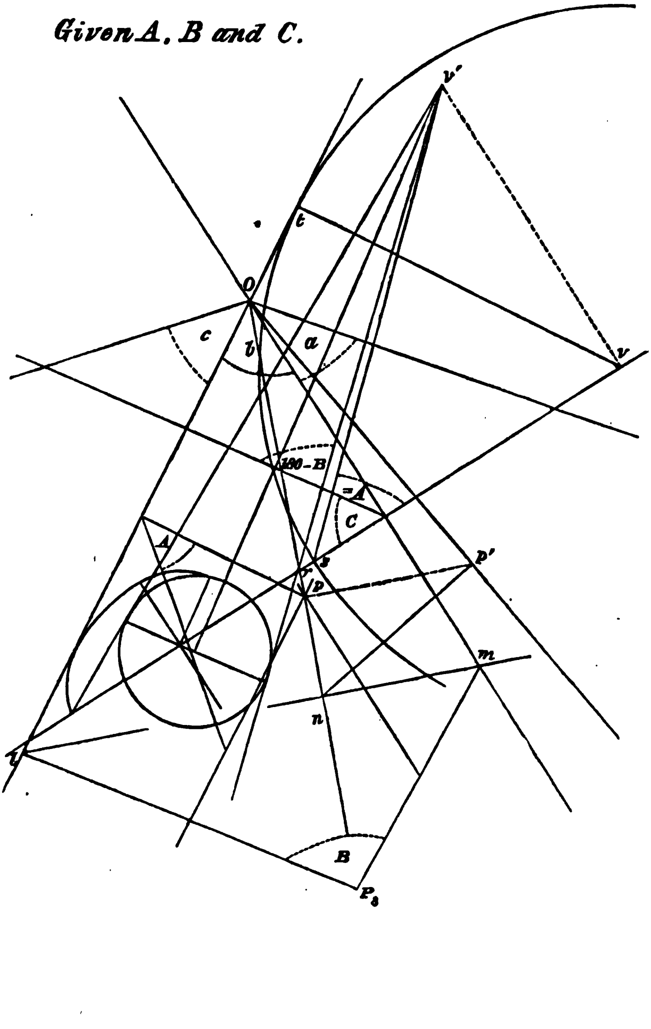
CASE VI.

Given the three angles A, B *and* C*: to determine the three sides* a, b *and* c.

If the dihedral angle C has one face in the horizontal plane, the traces of the other face can be readily drawn; and of these the horizontal trace is an indefinite edge of the required spherical triangle.

A third plane making the angle of A with its horizontal, and of $180 - B$ with the inclined plane, determined by Problem 26, Chap. III., will complete the required spherical triangle. The intersections Ol, OP of this third plane with the planes of the angle C form the second and third edges of the spherical triangle. The face b will thus be determined in the horizontal plane; into which the remaining faces a and c can be 'constructed' about their respective edges or horizontal traces in the face b.

CASE VI.



XIII.

ISOMETRIC PROJECTION.

In Isometric Projection solid bodies the chief planes of which are mutually at right angles are so represented, that from one drawing, *a plan*, the principal measurements as to length, breadth, and height can be obtained.

The solid, the cube for instance, is to be so placed that the three planes which meet together at one corner are equally inclined to the horizontal plane, the plane of projection. The three lines which meet at that corner will then be projected so as to form three equal angles of 120° each, and will be the plans of the three edges of the cube. The plans of the opposite edges will be parallels to these, and hence it follows that in Isometric Projection all angles which are in reality right angles are projected into angles of either 120° or 60°.

Commence therefore an Isometric problem by drawing three lines, making angles of 120° with one another, for the plans of the three lowest edges of the solid.

Then construct an Isometric scale, so as to be able to find the Isometric lengths of the required edges; measure these, when found, on the lines drawn for their plans, and complete the problem by parallels.

To construct an Isometric scale. It is known that the relation of a line to its Isometric projection is as $\sqrt{3}$ to $\sqrt{2}$. Two lines must therefore be drawn in this relation. For this purpose take any line, say an inch, and at one end of it, erect a perpendicular of the same length, and join the ends. The square on the hypothenuse of this right-angled triangle will be equal to the sum of the squares on the two sides containing the right angle, i. e. in this case to 2 square inches, wherefore the hypothenuse itself represents $\sqrt{2}$. If from one end of this line representing $\sqrt{2}$ a perpendicular of 1 inch is erected, and this triangle completed, this hypothenuse by similar reasoning will represent $\sqrt{3}$. We now have two lines representing $\sqrt{2}$ and $\sqrt{3}$ in required relation for the scale, and if the real lengths of edges be measured along $\sqrt{3}$ and perpendiculars dropped from their ends to $\sqrt{2}$, the parts thereby intercepted will be the Isometric plans of the required edges.

I.

Isometric Proiection of a brick, 9 *by* 4·5 *by* 3 *inches, to half scale.*

II.

Your instrument case, with the lid (1) *half open, and* (2) *open to its fullest extent, to a convenient scale.*

III.

Your T *square, to a convenient scale.*

IV.

To determine the Isometric Projection of a given circle.

(1) Describe the circle and circumscribe it with a square.

(2) Join the diagonals of this square. Join also the points where these diagonals cut the circle, thus inscribing a square in the circle.

(3) Determine the Isometric Projection of this plane figure, i.e. of these two squares.

(4) Then the required Isometric Projection of the given circle will be the ellipse drawn passing through the centres of the projections of the four sides of the larger or circumscribed square—and will have the Isometric projections of the diagonals of the smaller or inscribed square for its major and minor axes.

NOTE. That the semi-axes of the ellipse and the Isometric-radius (as it is termed, i.e. the line from the centre to a tangent-point of the curve) are in the proportion of $\sqrt{3} : \sqrt{1} : \sqrt{2}$ to one another.

V.

A square block, edge 4 inches, thickness 1 inch, with a circular hole, diameter 3 inches, through the centre. Give the Isometric scale.

VI.

The arches of a viaduct are 40 feet wide and semicircular. The piers are 6 feet thick, and 40 feet high to the springing of the arch. The width of the viaduct is 25 feet.

Scale 10 feet to 1 inch.

NOTE ON THE PROJECTION OF SHADOWS.

Light travels in straight lines, called *rays*, which are parallel, divergent, or convergent, according to the source from whence the light proceeds and the media through which it has passed.

The sun's rays are sensibly parallel; rays from the electric light are divergent, and, for all practical purposes, may be considered as diverging from a point; while rays of light which have passed through a double convex lens converge towards a point or *focus*.

If an opaque body be placed between a source of light and the object or objects illuminated, some of the rays will be received upon the interposed body and will, consequently, be cut off from illuminating certain parts of the objects beyond. Popularly the opaque body would be said to throw or cast a shadow. There is, however, an ambiguity in the term shadow as commonly employed, which it is convenient at times, when treating of the projection of shadows, to avoid by suitable nomenclature.

A moment's consideration of the subject will suffice to show that when the light from a given source is partly intercepted by an opaque body, there is carved as it were out of the surrounding bundle of luminous rays a well-defined core of darkness, the surface of which derives its configuration from the interposed body. To the surface which separates this core from the surrounding rays of light, the term *shadow-surface* will be given. The line of intersection of the shadow-surface with the plane or objects it meets, is plainly the boundary of the shadow cast upon those objects by the interposed body, and, in order to avoid unnecessary circumlocution when referring to this line, it will be denominated the *shadow-trace.*

From these considerations it is apparent that the problem for the student in any given case, is to find the shadow-surface and its intersection with the surfaces on which the shadow is cast. Sometimes the shadow-surface is treated as the boundary of a definite solid, at others a convenient number of lines are drawn in the surface, and their intersections found with the plane or objects on which the shadow falls; but in both cases the problem is one of the "Intersection of Solids," or of "Sections by Oblique Planes."

It must be noted that the shadows we deal with are purely *geometrical* and their boundaries or *traces* are sharp distinct lines. Those actually seen in nature, called *physical shadows*, are more or less hazy about the region of the shadow-trace, due partly to light diffused from surrounding objects and partly to the formation of a *penumbra*—a fringing semi-shadow, which makes its appearance when the source of light has any appreciable angular magnitude.

I. *Shadows cast by parallel rays.*

The rays of light are in all cases represented by real physical lines.

One of the parallel rays being given by its plan and elevation, the shadow-surface is supposed everywhere parallel to the given ray and enveloping the solid whose shadow is to be projected.

If the solid is a sphere, the shadow-surface is the enveloping cylinder whose axis is a line drawn from the centre of the sphere parallel to given ray. The shadow-trace is the intersection of this cylinder with the plane or objects on which the shadow of the sphere is cast.

If in lieu of the sphere we assume a right cone in space with axis oblique to co-ordinate planes, we may determine its shadow-surface thus:—Draw a line from vertex of cone parallel to given ray and determine two tangent planes to conic surface containing this line. The planes so drawn form a portion of the shadow-surface required, and, if the plane of the base of the cone be parallel to the given ray, that plane produced will intersect the two tangent planes in parallel lines giving rise to a triangular prism which will be the complete shadow-surface of the cone. The intersection of this prism with the plane or objects on which the shadow falls will give the shadow-trace as in other cases.

If the plane of the base be not parallel to the given ray, suitable points in that portion of it which assists in the determination of the shadow-surface may be taken, and lines drawn through them parallel to the given ray. The points of intersection of these lines with the objects they meet are points in the shadow-trace. Generally, proper points being assumed in any body whatever, and lines being drawn therefrom parallel to given ray, the line or curve joining the points of intersection of the parallel lines with the plane or curved surfaces on which the shadow is cast determines the shadow-trace. In many cases a few points suffice. For example, in finding the shadow cast on the horizontal plane by a cube, it will be seen that the shadow-surface is bounded by planes parallel to the given ray containing certain edges of the solid, and that the *shadow-trace* is made up of parts of the horizontal traces of these planes. If therefore two lines be drawn from the extremities of one of these edges parallel to the given ray and their horizontal traces found and joined by a straight line, this line will be one edge of the shadow-trace.

For practice the student can show the shadow-trace of a cone on a sphere, a cylinder, a tetrahedron, cube, &c. (1) when the cone stands on its base, (2) when lying on its slant side, and (3) when its axis is oblique to both planes of projection.

II. *Shadows cast by rays which meet in a point.*

In this division we include the projection of shadows cast by rays both *divergent* and *convergent.* The former is the only case we shall notice inasmuch as whatever is said concerning the one is, with very obvious modifications, equally applicable to the other.

The source of light is a point, given by its projections, from which the rays diverge in every possible direction.

The shadow-surface is generated by a line passing through the given point and moved round the body whose shadow is to be projected.

Thus, the shadow-surface of a sphere is an enveloping cone having its vertex in the given point. The shadow-trace will be determined in any given case by the methods specially applicable to the intersection of a cone and the particular solid or plane on which the shadow is cast.

If the cone selected as an example under parallel rays be taken, the work is substantially the same as there given. The two tangent planes to the cone which pass through the given luminous point will constitute a portion of the shadow-surface. *Any* required points in the shadow-trace can always be found by lines from the given luminous point touching the conic surface and produced to meet the objects on which the shadow-trace falls.

The shadow-surface of a solid bounded wholly by plane figures will be a more or less irregular pyramid having its vertex in the luminous point. The plane faces of this pyramid will contain certain of the edges of the solid whose shadow has to be projected, and in this case, as in the case with parallel rays, the work may sometimes require that two lines only be drawn from the point—one through each extremity of an edge—to determine one side of the shadow-trace.

As examples for practice the shadow of a sphere may be cast on a cone, another sphere, a cylinder or a pyramid.

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